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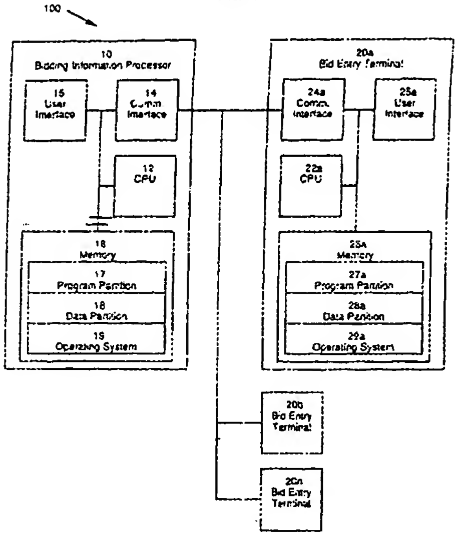
(54) System and method for an efficient dynamic multi-unit auction

(57) 1. A method for using a computer to implement an auction of heterogeneous objects two or more bidders participating in the auction, the auction allowing assignment of objects of the same type to one or more of the bidders at different prices, the method comprising:

a) inputting, into the computer, bids from participating bidders;
b) determining at the computer, based on the bids, whether there is at least one object which is desired by only one bidder and,
c) in the event there is at least one object which is desired by only one bidder, assigning the determined object or objects to the determined bidder,

wherein said determining comprises determining if a nonempty set of objects is minimal in the winning set of a bidder.

FIG. 1



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Description

Field of the Invention

[0001] The present invention relates to improving computer-implemented auctions and, more particularly, to computer implementation of an efficient dynamic multi-unit auction.

Background of the Invention

[0002] Auction formats in the art tend generally to be of the sealed-bid or ascending-bid variety. In the standard sealed-bid auction, bidders—in one single bidding round—simultaneously and independently submit bids to the auctioneer, who then determines the auction outcome. In the standard ascending-bid auction, bidders—in a dynamic bidding process—submit bids in real time until no more bids are forthcoming. An ascending-bid format offers the advantage that there is feedback between participants' bids: each bidder is able to infer other bidders' information about the value of the object(s) as the auction progresses and incorporate this information into his subsequent bids. This feedback tends to result in more efficient auction outcomes as well as more aggressive bidding, resulting in higher expected revenues for the seller.

[0003] However, standard ascending-bid formats—such as the design used by the Federal Communication Commission for auctioning radio communications spectrum—have the disadvantage that they do not generally lead to outcomes which are efficient in the sense of assigning objects to the bidders who value them the most. Most ascending-bid auction formats have the unfortunate property that identical objects sell at the uniform price reached at the end of the auction. This creates incentives for bidders to engage in demand reduction: bidders have incentive to understate the values that they place on marginal units in order to reduce the market-clearing price (and, hence, the price they will pay on the inframarginal units that they will win in any case). This has clear negative implications both for efficiency and for revenues.

[0004] My prior patent, "System and Method for an Efficient Dynamic Auction for Multiple Objects," (U.S. Patent Number 6,026,383, issued 15 Feb 2000) provides an early version of a system and method for a dynamic auctions which may achieve efficiency for situations involving multiple identical objects. The current invention is an improved system and method for a dynamic multi-unit auction which may achieve efficiency in more general economic environments.

Summary of the Invention

[0005] The present invention is a system and method for implementing on a computer a dynamic multi-unit auction in which the price paid or received by bidders tends to be independent of their own bids, in which participants may be provided with information concerning their competitors' bids as the auction progresses, and in which the confidentiality of high values is maintained. This provides the advantage of improving the economic efficiency of the auction design over the prior art. The present invention usefully enables a seller or buyer to efficiently auction multiple types of goods or services, and to efficiently auction items with complex possibilities for substitution.

[0006] The present invention comprises a computer that receives bids in a dynamic bidding process and assigns the items to bidders, and a method for receiving bids in a dynamic bidding process and assigning the items to bidders. In one embodiment, the invention comprises a bidding information processor (BIP) and a plurality of bid entry terminals (BET's) communicatively coupled to the bidding information processor. Bidders at the bid entry terminals enter bids in multiple rounds, and may observe displayed auction information. The bidding information processor and the bid entry terminals communicate and process information in order to conduct an auction.

[0007] Suppose that m ($m \geq 1$) types of objects are being auctioned, and one or more units of each type are being auctioned. An auction in accordance with an embodiment of the present invention proceeds as follows. First, the auctioneer (i.e., the bidding information processor) determines a starting price vector, $(P_1,...,P_m)$, and transmits it to bidders (i.e., bid entry terminals). Second, a bidder responds with a bid vector indicating the quantity of each respective type of object that the bidder wishes to transact at the current price vector. Let the bidders be superscripted by i, where $i = 1,...,n$. The bid vector for bidder i is denoted by $(Q_1^i,...,Q_m^i)$. Also, let the quantities of the respective types of objects being auctioned be denoted by $(\bar{Q}_1,...,\bar{Q}_m)$. Typically, the aggregate quantity of each type of object desired by all the bidders (i.e., $\sum_{i=1}^{n} Q_k^i$) is greater than the quantity of each type of object being auctioned (i.e., $\bar{Q}_k$). In this event, the auctioneer still determines whether any of the objects should be assigned to any bidders in this round. This is done by determining for each bidder, separately, whether the sum of the quantities bid by all the other bidders for all m types of objects is less than the sum of the quantities of all m types of objects being auctioned. In other words, there is at least one object which is desired by only one bidder. In the event that the auctioneer determines a bidder who should be assigned objects, the auctioneer further determines which type(s) of objects should be assigned to such bidder. This is done by determining for each type of object, separately, whether the sum of the quantities bid for this type of object by all the other bidders is less than the sum of the quantities being auctioned. In other words, there is at least one object of this type which is desired by only one bidder. Those objects, of those types, are then assigned to that bidder, obligating that bidder to transact them at the prices standing for those types of objects at that time. (If more than one possible assignment vector to that bidder is consistent with this rule, then the auctioneer is permitted to select his most-preferred assignment vector consistent with this rule.) If any objects remain unassigned, the auctioneer announces a new price vector and the auction continues.

[0008] Certain constraints are desirable in order for this auction to operate optimally and to reach an economically efficient outcome. One exemplary constraint is an activity rule which constrains a bidder not to increase his quantity, summed over the *m* types of objects, from one round to the next. Another exemplary constraint is a more stringent activity rule which constrains a bidder not to increase his quantity, individually on each of the *m* types of objects, from one bid or one round to the next. A third exemplary constraint is a reduction rule which constrains a bidder not to decrease his quantity, for any single type of object, beyond the point where the sum of the quantities bid for this type of object by all bidders equals the sum of the quantities being auctioned. (If, in a given round, two or more bidders simultaneously attempt to decrease their quantities, for any single type of object, having the effect of reducing bids beyond the point where the sum of the quantities bid for this type of object by all bidders equals the sum of the quantities being auctioned, the auction procedure will resolve this discrepancy. For example, the auctioneer may honor these attempts to decrease in order of time priority, or may ration these simultaneous attempts to decrease in proportion to the attempted reductions.)

[0009] While an auction following these rules could be conducted manually, computerized conduct of the auction allows the auction to be conducted with all bidding information taken into account, while controlling the degree to which the information itself is disclosed to the participants. Computerized conduct of the auction also allows the auction to be conducted swiftly and reliably, even if bidders are not located on-site. The amount of information which is transmitted to the bid entry terminals and/or actually displayed to the bidders may be carefully controlled. In one embodiment, all bidding information is displayed to the bidders. In another embodiment, no bidding information is displayed to the bidders; only the results of the auction are displayed. A number of intermediate embodiments are also possible, in which some but not all bidding information is displayed to the bidders. For example, in one preferred embodiment, the auctioneer disclose only the aggregate quantity bid for each type of object in each round, as opposed to disclosing each individual bid.

[0010] My prior patent 6,026,383 treats auctions for multiple, identical objects and close substitutes. The earlier application's alternative auction—which may be viewed as a special case of the current auction design—exploited features of the homogeneous-good environment to construct an eminently-simple dynamic procedure. Unfortunately, the cases of multiple types of objects, or objects with complex possibilities for substitution, do not lend themselves to quite as simple a procedure. My other prior patents, "Computer Implemented Methods and Apparatus for Auctions," U.S. Patent Number 5,905,975, issued 18 May 1999, and U.S. Patent Number 6,021,398, issued 1 Feb 2000, describe other auction designs for multiple, dissimilar objects. However, the current auction design appears likely in practice to be simpler and to run more swiftly, as well as placing lower computational demands on bidders.

[0011] The present invention generalizes my auction design described in U.S. patent 6,026,383 to treat—in a simple way—the case of auctioning a set of items which includes two (or more) items that are neither identical nor perfect substitutes to one another. Henceforth, this will be described for short as a situation with "multiple types of multiple objects," or simply "heterogeneous items" or "heterogeneous objects." Often, but not always, the heterogeneous items auctioned together will bear some relationship to one another: for example, they may be licenses or rights to perform essentially the same activity at different geographic locations; or they may be securities issued by the same entity but with different durations to maturity; or they may be related goods with slightly different characteristics that render them only imperfect substitutes.

[0012] The present invention may also be better suited than previous auction designs for treating the case of identical objects or perfect substitutes which exhibit "increasing returns" for bidders. "Increasing returns" refers to a situation where the extra value that a bidder derives from an $(N+1)^{st}$ unit is greater than the extra value that a bidder derives from an N^{th} unit. For example, this would include a situation where the utility from two units is strictly more than double the utility derived from one unit.

[0013] The present invention is useful for conducting auctions involving objects offered for sale by the bidders, as well as objects offered for sale to the bidders. Although terms such as "vector of quantities demanded" (by a bidder) and "demand curve" (of a bidder) are used to describe the present invention, the terms "vector of quantities offered" (by a bidder) and "supply curve" (of a bidder) are equally applicable. In some cases, this is made explicit by the use of both terms, or by the use of the terms "vector of quantities transacted" (by a bidder) and "transaction curve" (of a bidder). The term "quantities transacted" includes both "quantities demanded" and "quantities offered". The term "bid" includes both offers to sell and offers to buy. The term "transaction curve" includes both "demand curve" and "supply curve". Moreover, any references to "quantities being offered" includes both "quantities being sold" by the auctioneer, in the case this is an auction for selling objects, as well as "quantities being bought or procured" by the auctioneer, in the case this is an auction for buying objects or procuring objects.

[0014] Moreover, while standard auctions to sell typically involve ascending prices, the present invention may utilize prices that ascend and/or descend. One useful situation in which the price would be allowed to descend is a procurement auction or "reverse auction," an auction to buy.

[0015] Throughout this document, the terms "objects", "items", "units" and "goods" are used essentially interchangeably. The inventive system may be used both for tangible objects, such as real or personal property, and intangible objects, such as telecommunications licenses or electric power. The inventive system may be used in auctions where the auctioneer is a seller, buyer or broker, the bidders are buyers, sellers or brokers, and for auction-like activities which cannot be interpreted as selling or buying. The inventive system may be used for items including, but not restricted to, the following: public-sector bonds, bills, notes, stocks, and other securities or derivatives; private-sector bonds, bills, notes, stocks, and other securities or derivatives; communication licenses and spectrum rights; clearing, relocation or other rights concerning encumbrances of spectrum licenses; electric power and other commodity items; rights for terminal, entry, exit or transmission capacities or other rights in gas pipeline systems; airport landing rights; emission allowances and pollution permits; and other goods, services, objects, items or other property, tangible or intangible. It may be used in initial public offerings, secondary offerings, and in secondary or resale markets.

[0016] The communication system used, if any, can be any system capable of providing the necessary communication and includes for example a local or wide area network such as for example ethernet, token ring, or alternatively a telephone system, either private or public, the Internet, the Worldwide Web or the information superhighway.

Brief Description of the Drawings

[0017]

Figure 1 is an exemplary block diagram of an auction system in accordance with the invention;
Figure 2 is a detail of one element of the system of Fig. 1;
Figure 3 is a flow diagram of an auction process in accordance with one embodiment of the invention;
Figures 4a, 4b, and 5 are more detailed flow diagrams illustrating, in more detail, elements of the diagram of fig. 3;
Figure 6 is a flow diagram of an auction process in accordance with another embodiment of the invention;
Figure 7 is a flow diagram of an auction process in accordance with another embodiment of the invention; and
Figures 8a, 8b, 8c and 9 are more detailed diagrams illustrating elements of an earlier diagram.

Detailed Description of Preferred Embodiments

[0018] The drawings of Figures 1-4 of my prior patent 6,026,383 and of Figures 1-12 of my patent 5,905,975, and the associated text, provide a general superstructure for the present auction method and system, especially as it relates to the computer implementation thereof. Moreover, the terminology established in the previous applications will be relied upon as needed. The following description will detail the flow of the novel features of the preferred embodiments of the present method and system for an efficient dynamic auction for multiple types of multiple objects.

[0019] Before describing how the auction process is implemented, reference is made to FIG. 1 to describe an exemplary block diagram of one embodiment of the a system in accordance with the present invention. As illustrated in FIG. 1, the auction system includes an auctioneer's system 10 (sometimes also referred to as a Bidding Information Processor or BIP) and a plurality of user systems 20a, 20b and so on (sometimes also referred to as Bid Entry Terminal or BET), each user system 20a, etc. represents an individual bidder. The systems 10-20n are communicatively interconnected via a communication system represented by the illustrated connections. The communication system can represent any system capable of providing the necessary communication to/from BIP and BET and includes for example a local or wide area network such as for example ethernet, token ring, or alternatively a telephone system, either private or public, the internet, the worldwide web or the information superhighway. Each of the systems 10-20n includes a typical user interface 15, 25a for input/output and can include a conventional keyboard, display, and other conventional I/O devices. Within each of the systems, the user interface (15, 25a, etc.) is coupled to a communication interface (14, 24a, etc.) which is in turn connected to the communication system. Both the user interface and communication interface are also connected, at each system, to a CPU (12, 22a, etc.). Each system includes a memory (16, 26a, etc.) which can further be broken down into a program partition (17, 27a, etc.), a data partition (18, 28a, etc.) and an operating system partition (19, 29a, etc.). In each system the CPU (12, 22a, etc.) represents a source of intelligence when executing instructions from the memory (16, 26a, etc.) so that appropriate input/output operations via the user interface and the communications interface take place as is conventional in the art. The particular steps used in implementing the inventive auction system are described in more detail below. In one embodiment, each of the systems are personal computers or workstations.

[0020] Figure 2 is a more detailed illustration of an exemplary BIP 10 showing details of the data partition 18. As seen in Fig. 2 the data partition includes provision for creating, storing, processing and outputting values representing Current Lot Number 18-1, Current Round Number 18-2, Current Price 18-3, List of Bidder Numbers 18-4, Bidding History 18-5, Constraints on Bids 18-6, Passwords, 18-7 and Current Objects Available 18-8. The particular set of data required for any particular auction and the format of that datum or data (such as scalar, vector, list, etc.) is more particularly specified by the detailed description of that auction.

Embodiments Concerned with Complex Possibilities for Substitution

[0021] Some of the simplest embodiments of the present invention apply in situations where the possibilities for substitution among the heterogeneous items can be expressed as a matrix. Henceforth, the term "requirements matrix" will refer to a matrix of rows and columns which expresses the possibilities for substitution among the heterogeneous goods, the seller(s) or the buyer(s). The same information could be equivalently expressed as a graph.

[0022] For example, one embodiment of the present invention has useful application in the allocation of capacity for a gas pipeline system. Let Types 1 - 4 denote four geographically-dispersed terminals where gas may enter the pipeline system. The party on whose behalf the auction is conducted may wish to purchase 100 units of entry rights in aggregate. However, it may be the case that: 40 of these units are required to be of Type 2, since 40 units of gas are produced at a field that is located in close geographic proximity to the terminal corresponding to Type 2; 35 of these units are required to be of Types 3 or 4, since 35 units of gas are produced at fields that are located in close geographic proximity to the terminals corresponding to Types 3 and 4; while the remaining 25 units can be of any Types 1 - 4, since 25 units of gas are brought in by ship and can be landed equally easily at any of these terminals. This situation is depicted by the requirements matrix in Table 1:

$$\begin{array}{rc} & \textit{Units required} \\ \begin{array}{r} \textit{Type 1} \\ \textit{Type 2} \\ \textit{Type 3} \\ \textit{Type 4} \end{array} & \begin{pmatrix} 0 & 0 & 25 \\ 40 & 0 & 25 \\ 0 & 35 & 25 \\ 0 & 35 & 25 \end{pmatrix} \end{array}$$

Table 1

[0023] A second embodiment of the invention treats a situation where there are three types of items ($m = 3$) —Type 1, Type 2 and Type 3—being sold. The party on whose behalf the auction is conducted may wish to sell fixed quantities of these three individual types of items. However, it may still be the case that it is advantageous to sell them together, since bidders may find: that information revealed in the auction about one type of these items is relevant to the value of another type of these items; or that they are relatively more interested in one type of these items, or another, depending upon their relative prices. This situation is depicted by the requirements matrix in Table 2:

$$\begin{array}{rc} & \textit{Units required} \\ \begin{array}{r} \textit{Type 1} \\ \textit{Type 2} \\ \textit{Type 3} \end{array} & \begin{pmatrix} 2000 & 0 & 0 \\ 0 & 2000 & 0 \\ 0 & 0 & 1000 \end{pmatrix} \end{array}$$

Table 2

[0024] One useful application of this second embodiment occurs in the context of selling Treasury or other securities. A government or central bank may wish to simultaneously auction 3-month, 6-month and 12-month Treasury securities. Let us denote these three types of Treasury securities as Type 1, Type 2 and Type 3, respectively. The requirements matrix of Table 2 would say that the government or central bank has 2000 units of 3-month Treasury securities, 2000 units of 6-month Treasury securities, and 1000 units of 12-month Treasury securities.

[0025] A third, and mathematically simpler, embodiment of the present invention treats the situation where two types of items ($m = 2$)—Type 1 and Type 2—are being procured. The party on whose behalf the auction is conducted may wish to purchase four units in aggregate. However, it may be the case that three of these units are required to be of Type 1, while the fourth unit can either be of Type 1 or Type 2. This situation is depicted by the requirements matrix in Table 3:

$$\begin{array}{rc} & \textit{Units required} \\ \begin{array}{r} \textit{Type 1} \\ \textit{Type 2} \end{array} & \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

Table 3

[0026] One useful application of this third embodiment of the invention may occur in the clearing or relocation of television stations. For example, to clear the communications spectrum currently occupied by UHF Channels 59 - 69 in a given regional market, it might be necessary to relocate three analog and one digital television stations to lower channels. Let Type 1 denote an analog television station and let Type 2 denote a digital television station. Suppose that an analog station in UHF Channels 59 - 69 must be relocated to a frequency allocated to analog stations below Channel 59, while a digital station in UHF Channels 59 - 69 may be relocated to a frequency allocated to *either* an analog *or* a digital station below Channel 59. Then the requirements matrix of Table 3 would accurately describe the possibilities for substitution. In particular, the party on whose behalf the auction is held would be satisfied by purchasing four items of Type 1 or by purchasing three items of Type 1 and one item of Type 2. Generally speaking, since this is a procurement auction, this party would prefer whichever of these purchases cost less money.

DEFINITIONS.

[0027] The available quantity $(\bar{Q}_1,\ldots,\bar{Q}_m)$ refers to the quantity of each type of item offered for sale in the auction, in the case of an auction to sell, and the quantity of each type of item offered to be procured in the auction, in the case of an auction to buy (i.e., a reverse auction). Optionally, the available quantity may be allowed to depend on the price, or otherwise be contingent on the progress of the auction.

[0028] The bidding state consists of the current bids $\{Q_1^i,\ldots,Q_m^i\}_{i=1}^n$ of all the bidders, the available quantity, and the quantity vectors (if any) that have already been awarded to bidders. Optionally, bids may be permitted to be more complicated than merely a vector of quantities: for example, bids may be permitted to include prices or to be contingent on events, to contain "and" and "or" restrictions, and to include minimum acceptable quantities. Bids may also be permitted to specify arbitrary sets, or combinations of sets, of specifically-identified objects, as opposed to merely comprising quantities of types of objects.

[0029] The set of feasible assignments given the bidding state is the set of all possible allocations $\{x_1^i,\ldots,x_m^i\}_{i=1}^n$ of the available quantity of the *m* types of items to the *n* bidders, subject to satisfying all the constraints on the assignment of the goods, the constraints on the bidding process and the constraints posed by the bidding state. The following conditions (i)-(iv) are exemplary of the requirements on $\{x_1^i,\ldots,x_m^i\}_{i=1}^n$ to be a feasible assignment:

(i) $x_k^i \geq 0$ for every $k = 1,\ldots,m$, and for every $i = 1,\ldots,n$;
(ii) $\sum_{i=1}^n x_k^i = \bar{Q}_k$;
(iii) $\{x_1^i,\ldots,x_m^i\}_{i=1}^n$ is consistent with the requirements matrix or other constraints on the assignment of the items;
(iv) $\{x_1^i,\ldots,x_m^i\}_{i=1}^n$ is consistent with activity rules constraining the bidding process, given current bids $\{Q_1^i,\ldots,Q_m^i\}_{i=1}^n$ (e.g., $x_k^i \leq Q_k^i$ for every $k = 1,\ldots,m$, and for every $i = 1,\ldots,n$).

[0030] The winning set of bidder *i* is the set W^i of all $(x_1^i,\ldots,x_m^i)$ that are part of a feasible assignment (i.e., the set of feasible assignments for all bidders, projected onto bidder *i*).

[0031] We will write that a quantity vector $(a_1,\ldots,a_m) \geq (b_1,\ldots,b_m)$, if $a_k \geq b_k$ for every $k = 1,\ldots,m$, and if $a_k > b_k$ for some $k = 1,\ldots,m$.

[0032] A quantity vector $(q_1^i,\ldots,q_m^i)$ is said to be minimal in the winning set of bidder *i* if $(q_1^i,\ldots,q_m^i) \in W^i$ but $(r_1^i,\ldots,r_m^i) \notin W^i$ whenever $(q_1^i,\ldots,q_m^i) \geq (r_1^i,\ldots,r_m^i)$.

[0033] A bidder *i is* said to have clinched a quantity vector $(q_1^i,\ldots,q_m^i)$ if, given the available quantity, his opponents' bids and the various constraints on the auction, the bidder is mathematically guaranteed to win at least the quantity vector $(q_1^i,\ldots,q_m^i)$. In the above notation, bidder *i* clinches a quantity vector $(q_1^i,\ldots,q_m^i)$ if $(q_1^i,\ldots,q_m^i) \geq 0$ and if $(q_1^i,\ldots,q_m^i)$ is minimal in the winning set of bidder *i*.

[0034] The definition of clinching can be restated in simpler form in general set notation. Let Ω denote the set of available objects. An assignment is defined to be an *n*-tuple of subsets, $\{S^i\}_{i=1}^n$, where S^i denotes the subset of objects assigned to bidder *i*. A feasible assignment is an *n*-tuple of subsets, $\{S^i\}_{i=1}^n$, with the properties that: $\{S^i\}_{i=1}^n$ assigns every available object to exactly one bidder (i.e., the set union of $\{S^i\}_{i=1}^n$ equals Ω, and the $\{S^i\}_{i=1}^n$ are pairwise disjoint); $\{S^i\}_{i=1}^n$ is consistent with the requirements matrix or other constraints on the assignment of the objects; and $\{S^i\}_{i=1}^n$ is consistent with any activity rules constraining the bidding process, given the current bidding state. The winning set of bidder *i* is the set W^i of all subsets of objects, S^i, that are assigned to bidder *i* in some feasible assignment (i.e., the set of feasible assignments for all bidders, projected onto bidder *i*). A subset S^i of objects is said to be minimal in the winning set of bidder *i* if $S^i \in W^i$, but $R^i \notin W^i$ whenever R^i is a strict subset of S^i (i.e., there is no strict subset of S^i that is feasible for bidder *i* to win). Analogous to the previous paragraph, bidder *i* clinches a subset S^i of objects if $S^i \neq \emptyset$ (i.e., S^i is a nonempty set of objects) and S^i is minimal in the winning set of bidder *i*.

[0035] A second, almost equivalent way of describing clinching is that the quantity vector $(q_1^i,\ldots,q_m^i)$ is clinched if it constitutes a maximal quantity vector that bidder *i* is guaranteed to win. The quantity vector $(q_1^i,\ldots,q_m^i)$ is a maximal quantity vector that bidder *i* is guaranteed to win if: bidder *i* is guaranteed to win $(q_1^i,\ldots,q_m^i)$; but there does not exist any quantity vector $(r_1^i,\ldots,r_m^i) \geq (q_1^i,\ldots,q_m^i)$ such that bidder *i* is guaranteed to win the quantity vector $(r_1^i,\ldots,r_m^i)$

[0036] A third, and apparently equivalent way of describing clinching is in terms of admissible fictitious bids. We will say that $(\tilde{Q}_1^i,\ldots,\tilde{Q}_m^i)$ is an admissible fictitious bid if the following three conditions are all satisfied:

(i) $0 \leq \tilde{Q}_k^i \leq Q_k^i$, for every $k = 1,\ldots,m$,
(ii) $\tilde{Q}_k^i > 0$, for some $k = 1,\ldots,m$, and
(iii) $\tilde{Q}^i$ satisfies any extra requirements of the auc-

tion such as consisting entirely of integer numbers.

For any Bidder *i* and for any fictitious bid $\hat{Q}^i$ under consideration, the computer determines an answer to the following question: "If Bidder *i*'s actual bid $(Q_1^i,...,Q_m^i)$ were replaced by the admissible fictitious bid $(\hat{Q}_1^i ,...,\hat{Q}_m^i)$, and if the other bidders continued to use their actual bids, would the auction conclude?" If the answer to this question is affirmative, then Bidder *i* can also be viewed as having guaranteed winning the units $(Q_1^i - \hat{Q}_1^i ,..., Q_m^i - \hat{Q}_m^i)$. The auction procedure takes note of this determination in assigning objects.

[0037] The principles underlying "clinching"—and the algorithm determining "clinching"—are easiest illustrated with some examples:

[0038] EXAMPLE 1. The requirements matrix for units of each of the two types is given by Table 3. The auction is conducted as a procurement auction, and four bidders (superscripted by *i* = 1,2,3,4) participate. At every price *p*, each bidder *i* indicates a pair (Q_1^i, Q_2^i), which gives the number of units of Type 1 and the number of units of Type 2, respectively, that he is willing to sell at price *p*. Suppose that, for a given *price p*, the following Table 4A shows the quantities that the bidders have indicated:

Table 4A

	Q_1^i	Q_2^i
Bidder 1	2	0
Bidder 2	1	0
Bidder 3	0	1
Bidder 4	1	0

To begin, observe that the auction has not yet concluded, and that the auctioneer may continue by naming a new, lower price. The reason for this observation is that the party on whose behalf the auction is being conducted only wishes to purchase either 3 units of Type 1 and 1 unit of Type 2, or 4 units of Type 1 (see Table 3), whereas Bidders 1 - 4 are in aggregate offering 4 units of Type 1 and 1 unit of Type 2 (i.e., strictly more than is required). However, suppose we take any Bidder *i* (*i* = 1,2,3,4) and consider the following question: "Are there any types of any units that Bidder *i* is already guaranteed to have won (given other bidders' bids)?" If the answer to this question is affirmative, then Bidder *i* will be said to have clinched such units.

[0039] In Example 1, let us begin by considering Bidder 1. Since Bidder 1 is only bidding on units of Type 1, Bidder 1 can only clinch units of Type 1. Observe that any feasible way of satisfying the requirements matrix includes Bidder 1 winning on at least 1 unit of Type 1. (This follows from the fact that, in the requirements matrix of Table 3, at least 3 units of Type 1 are required, but Bidders 2 - 4 are collectively bidding only 2 units of Type 1.) Thus, Bidder 1 has clinched one unit of Type 1. One way that the auctioneer may act on this determination is to assign one unit of Type 1 to Bidder 1 at the price associated with the first time that Bidder 1 was determined to have clinched one unit of Type 1.

[0040] At the same time, observe that one feasible way of satisfying the requirements matrix is using the following bids: (Q_1^1, Q_2^1) = (1,0); (Q_1^2, Q_2^2) = (1,0); (Q_1^3, Q_2^3) = (0,1); and (Q_1^4, Q_2^4) = (1,0). Since this feasible way of satisfying the requirements matrix has Bidder 1 winning only one unit of Type 1, we conclude that Bidder 1 has not clinched two units of Type 1.

[0041] To put things slightly differently, but equivalently, the winning set for Bidder 1 in Example 1 is W^1 = {(1,0), (2,0)}. There is no feasible assignment in which Bidder 1 is assigned (0,0). Thus, (1,0) is minimal in the winning set, W^1, of Bidder 1, and so Bidder 1 clinches the quantity vector (1,0). Put differently, if Bidder 1's actual bid of (2,0) were replaced by the admissible fictitious bid (1,0), then the auction would end; thus, Bidder 1 clinches the difference (2,0)- (1,0) = (1,0).

[0042] Also at the same time, observe that Bidders 2 - 4 have not yet clinched any units. The requirements matrix can be satisfied using the bids: (Q_1^1, Q_2^1) = (2,0); (Q_1^2, Q_2^2) = (0,0); (Q_1^3, Q_2^3) = (0,1); and (Q_1^4, Q_2^4) = (1,0). This does not involve Bidder 2 winning anything at all. The requirements matrix can also be satisfied using the bids: (Q_1^1, Q_2^1) = (2,0); (Q_1^2, Q_2^2) = (1,0); (Q_1^3, Q_2^3) = (0,0); and (Q_1^4, Q_2^4) = (1,0). This does not involve Bidder 3 winning anything at all. Finally, the requirements matrix can also be satisfied using the bids: (Q_1^1, Q_2^1) = (2,0); (Q_1^2, Q_2^2) = (1,0); (Q_1^3, Q_2^3) = (0,1); and (Q_1^4, Q_2^4) = (0,0). This does not involve Bidder 4 winning anything at all.

[0043] EXAMPLE 2. The requirements matrix for units of each of the two types is again given by Table 3. The auction is conducted as a procurement auction, and four bidders (superscripted by *i* = 1,2,3,4) participate. At every price *p*, each bidder *i* indicates a pair (Q_1^i, Q_2^i), which gives the number of units of Type 1 and the number of units of Type 2, respectively, that he is willing to sell at price *p*. Suppose that, for a given price *p*, the following Table 4B shows the quantities that the bidders have indicated:

Table 4B

	Q_1^i	Q_2^i
Bidder 1	1	1
Bidder 2	1	0
Bidder 3	0	1
Bidder 4	1	0

To begin, observe that the auction has not yet concluded, and that the auctioneer may continue by naming a new, lower price. The reason for this observation is that the party on whose behalf the auction is being conducted only wishes to purchase either 3 units of Type 1 and 1 unit of Type 2, or 4 units of Type 1 (see Table 3), whereas Bidders 1 - 4 are in aggregate offering 3 units of Type 1 and 2 units of Type 2 (i.e., strictly more than is required).

[0044] In Example 2, let us begin by considering Bidder 1. Since Bidder 1 is bidding on units of Type 1 and Type 2, Bidder 1 can in principle clinch units of Type 1 or Type 2. Observe that any feasible way of satisfying the requirements matrix includes Bidder 1 winning 1 unit of Type 1. (This follows from the fact that, in the requirements matrix of Table 3, at least 3 units of Type 1 are required, but Bidders 2 - 4 are collectively bidding only 2 units of Type 1.) Thus, Bidder 1 has clinched one unit of Type 1. One way that the auctioneer may act on this determination is to assign one unit of Type 1 to Bidder 1 at the price associated with the first time that Bidder 1 was determined to have clinched one unit of Type 1.

[0045] Bidders 2 and 4 have also clinched one unit of Type 1. (For Bidder 2, this follows from the fact that, in the requirements matrix of Table 3, at least 3 units of Type 1 are required, but Bidders 1, 3 and 4 are collectively bidding only 2 units of Type 1.) (For Bidder 4, this follows from the fact that, in the requirements matrix of Table 3, at least 3 units of Type 1 are required, but Bidders 1, 2 and 3 are collectively bidding only 2 units of Type 1.) One way that the auctioneer may act on this determination is to assign one unit of Type 1 to Bidder 2 at the price associated with the first time that Bidder 2 was determined to have clinched one unit of Type 1, and similarly for Bidder 4.

[0046] At the same time, observe that Bidder 1 has not clinched one unit of Type 2. This is because one feasible way of satisfying the requirements matrix is using the following bids: $(Q_1^1, Q_2^1) = (1,0)$; $(Q_1^2, Q_2^2) = (1,0)$; $(Q_1^3, Q_2^3) = (0,1)$; and $(Q_1^4, Q_2^4) = (1,0)$. Since this feasible way of satisfying the requirements matrix has Bidder 1 winning zero units of Type 2, we conclude that Bidder 1 has not clinched any units of Type 2.

[0047] Also at the same time, observe that Bidder 3 has not yet clinched any units. The requirements matrix can be satisfied using the bids: $(Q_1^1, Q_2^1) = (1,1)$; $(Q_1^2, Q_2^2) = (1,0)$; $(Q_1^3, Q_2^3) = (0,0)$; and $(Q_1^4, Q_2^4) = (1,0)$. This does not involve Bidder 3 winning anything at all.

[0048] EXAMPLE 3. Let the requirements matrix for units of each of three types now be given by Table 2. The auction is conducted as a selling auction, and four bidders (superscripted by i= 1,2,3,4) participate. At every price vector (p_1,p_2,p_3), each bidder i indicates a vector (Q_1^i, Q_2^i, Q_3^i), which gives the number of units of Type 1 that he is willing to purchase at price p_1, the number of units of Type 2 that he is willing to purchase at price p_2, and the number of units of Type 3 that he is willing to purchase at price p_3. Suppose that, for a given price vector (p_1, p_2, p_3), the following Table 5 shows the quantities that the bidders have indicated:

Table 5

	Q_1i	Q_2^i	Q_3i
Bidder 1	1000	0	500
Bidder 2	500	1000	100
Bidder 3	500	1000	200
Bidder 4	500	0	600

We might also suppose that, as the auction continues, each bidder is only allowed to bid the same quantity or lower on each type of item.

[0049] To begin, observe that the auction has not yet concluded on Type 1 and Type 3, and that the auctioneer may continue by naming a new, higher prices p_1 and p_3. The reason for this observation is that the party on whose behalf the auction is being conducted only wishes to sell 2000 units of Type 1, whereas Bidders 1 - 4 are in aggregate demanding 2500 units of Type 1 (i.e., strictly more than is required), and the party on whose behalf the auction is being conducted only wishes to sell 1000 units of Type 3, whereas Bidders 1 - 4 are in aggregate demanding 1400 units of Type 3 (i.e., strictly more than is required).

[0050] In Example 3, since the requirements matrix is a diagonal matrix, one may choose to treat the three types of units separately. Let us begin by considering Type 1. In considering Type 1, let us begin with Bidder 1. Observe that any feasible way of satisfying the requirements matrix includes Bidder 1 winning (at least) 500 units of Type 1. (This follows from the fact that, in the requirements matrix of Table 5, there are 2000 units of Type 1 required, but Bidders 2 - 4 are collectively bidding only 1500 units of Type 1.) Thus, Bidder 1 has clinched 500 units of Type 1. One way that the auctioneer may act on this determination is to assign units of Type 1 to Bidder 1 at the price associated with the first time that Bidder 1 was determined to have clinched the given unit of Type 1.

[0051] At the same time, observe that Bidders 2 - 4 have not yet clinched any units of Type 1. The requirements for Type 1 can be satisfied using the quantities: Q_1^1 = 1000; Q_1^2 = 0; Q_1^3 = 500; and Q_1^4 = 500. This does not involve Bidder 2 winning any of Type 1 at all. The requirements for Type 1 can also be satisfied using the quantities: Q_1^1 = 1000; Q_1^2 = 500; Q_1^3 = 0; and Q_1^4 = 500. This does not involve Bidder 3 winning any of Type 1 at all. The requirements for Type 1 can also be satisfied using the quantities: Q_1^1 = 1000; Q_1^2 = 500; Q_1^3 = 500; and Q_1^4 = 0. This does not involve Bidder 4 winning any of Type 1 at all.

[0052] Turning to Type 2, observe that the auction of Type 2 items has effectively concluded. In clinching terms, Bidders 2 and 3 have each clinched 1000 units of Type 2, and their combined clinches equals the entire requirements of 2000. This follows from the fact that the only feasible way in which the requirements for Type 2 can be satisfied is using the quantities: $Q_2^1 = 0$; $Q_2^2 = 1000$; $Q_2^3 = 1000$; and $Q_2^4 = 0$.

[0053] Turning to Type 3, let us begin by considering Bidder 1. Observe that any feasible way of satisfying the requirements matrix includes Bidder 1 winning (at least) 100 units of Type 3. (This follows from the fact that, in the requirements matrix of Table 2, there are 1000 units of Type 3 required, but Bidders 2 - 4 are collectively bidding only 900 units of Type 3.) Thus, Bidder 1 has clinched 100 units of Type 3. One way that the auctioneer may act on this determination is to assign units of Type 3 to Bidder 1 at the price associated with the first time that Bidder 1 was determined to have clinched the given unit of Type 3. Let us next consider Bidder 4. Observe that any feasible way of satisfying the requirements matrix includes Bidder 4 winning (at least) 200 units of Type 3. (This follows from the fact that, in the requirements matrix of Table 2, there are 1000 units of Type 3 required, but Bidders 1 - 3 are collectively bidding only 800 units of Type 3.) Thus, Bidder 4 has clinched 200 units of Type 3. One way that the auctioneer may act on this determination is to assign units of Type 3 to Bidder 4 at the price associated with the first time that Bidder 4 was determined to have clinched the given unit of Type 3.

[0054] At the same time, observe that Bidders 2 and 3 have not clinched any units of Type 3. This is because one feasible way in which the requirements for Type 3 can be satisfied is using the quantities: $Q_3^1 = 400$; $Q_3^2 = 0$; $Q_3^3 = 0$; and $Q_3^4 = 600$. This does not involve Bidder 2 or Bidder 3 winning any of Type 3 at all.

[0055] Figure 3 is a flow diagram of an auction process in accordance with one embodiment of the present invention. The process starts with step 202, in which memory locations at the computer are initialized. In step 202, the appropriate memory locations are initialized with information such as the number of types of objects for auction, the quantity of each type of object for auction, and the initial price vector. In step 204, the computer outputs auction information, including the current price vector $(P_1,\ldots,P_m)$. In step 206, the computer receives bids $(Q_1^i,\ldots,Q_m^i)$ from bidders. In step 208, the computer closes the bidding for the current round and processes bids. This process is shown in more detail in Figure 5. In step 210, the computer determines whether: (a) there exist zero feasible assignments of the available quantity; (b) there exists one feasible assignment of the available quantity; or (c) there exist two or more feasible assignments of the available quantity. If, at step 210, the computer determines that (c) there exist two or more feasible assignments of the available quantity, then the process goes to step 216 in which the computer determines whether any units have been clinched by any bidders and, if so, assigns clinched units to determined bidders at current prices. This process is shown in more detail in Figure 4a, and one preferred embodiment of this process is shown in more detail in Figure 4b. The process then goes to step 218 in which the computer revises the current price vector $(P_1,\ldots,P_m)$ and generates the bidding history and any auction announcements and messages. One exemplary rule for revising the price is that, for every $k = 1,\ldots,m$, the price P_k of objects of type k is raised by $c[(\sum_{i=1}^{n} Q_k^i)-\overline{Q}_k^t]$, where c is a positive constant (i.e., the price for each type is increased in direct proportion to the excess demand for that type). The process then loops to step 204.

[0056] If, at step 210, the computer determines that (a) there exist zero feasible assignments of the available quantity, then the process goes to step 212 in which the computer assigns units in accordance with a rationing rule. One exemplary rationing rule is for the computer to honor the various bidders' attempts to decrease their bids by time priority (i.e., in the order that the bids were submitted) so long as a feasible assignment continues to exist. A second exemplary rationing rule is for the computer to increase each bidder's quantity bid in constant proportion to the bidder's most recent attempted reduction, $Q_k^{i,t-1} - Q_k^{i,t}$, until one assignment becomes feasible. If, at step 210, the computer determines that (b) there exists one feasible assignment of the available quantity, then the process goes to step 214 in which the computer assigns units in accordance with the one feasible assignment. After steps 212 or 214, the auction is deemed to have ended, and so the process ends.

[0057] Figure 4a is a flow diagram of a subprocess of step 216. It begins with step 216a-1, in which a bidder i which has not yet been considered is selected. In step 216a-2, for the bidder i currently being considered, the computer determines whether there exists a quantity vector $(q_1^i,\ldots,q_m^i) \geq 0$ that bidder i is guaranteed to win. If there does not exist such a quantity vector $(q_1^i,\ldots,q_m^i) \geq 0$, the process proceeds directly to step 216a-7. If there does exist such a quantity vector $(q_1^i,\ldots,q_m^i) \geq 0$, the process continues with step 216a-3, in which the computer determines a maximal quantity vector $(q_1^i,\ldots,q_m^i) \geq 0$ that bidder i is guaranteed to win. The quantity vector $(q_1^i,\ldots,q_m^i)$ is said to be a maximal quantity vector that bidder i is guaranteed to win (or a maximal guaranteed quantity vector) if: bidder i is guaranteed to win $(q_1^i,\ldots,q_m^i)$; but there does not exist any quantity vector $(r_1^i,\ldots,r_m^i) \geq (q_1^i,\ldots,q_m^i)$ such that bidder i is guaranteed to win the quantity vector $(r_1^i,\ldots,r_m^i)$. The process then continues with step 216a-4, in which the computer determines whether the maximal guaranteed quantity vector $(q_1^i,\ldots,q_m^i) \geq 0$ generated in step 216a-3 is unique. If it is unique, the process proceeds directly to step 216a-6. If it is not unique, the process continues with step 216a-5, in which the computer determines a most-preferred maximal quantity vector $(q_1^i,\ldots,q_m^i)$ that bidder i is guaranteed to win. One exemplary way of selecting a most-preferred maximal guaranteed quantity vector that bidder i is guaranteed to win is to calculate a maximal guaranteed quantity vector $(q_1^i,\ldots,q_m^i)$ such that the difference between the expected final price vector and the current price vector, multiplied by $(q_1^i,\ldots,q_m^i)$ as a dot product, is minimized. In step 216a-6, the computer assigns the determined quantity vector, $(q_1^i,\ldots,q_m^i)$, to bidder i at the current price vector $(P_1,\ldots,P_m)$. In step 216a-7, the computer determines whether all bidders have been considered. If not, the process loops back to step 216a-1. If all bidders have been considered, the process goes to step 218 of Figure 3.

[0058] Figure 4b is a flow diagram of one preferred embodiment of the process shown in Figure 4a. It is also a flow diagram of a subprocess of step 216. It begins with step 216b-1, in which the computer selects a bidder i which has not yet been considered. In step 216b-2, the computer selects a type k which has not yet been considered. In step 216b-3, the computer sums the quantities demanded of the type k objects by all the bidders except bidder i, and compares this sum with the quantity remaining unassigned of type k objects. If the computer determines that the quantity remaining unassigned of type k objects is not strictly greater than the sum of the quantities demanded of the type k objects by all the bidders except bidder i, then the process proceeds directly to step 216b-5. If the computer determines that the quantity remaining unassigned of type k objects is strictly greater than the sum of the quantities demanded of the type k objects by all the bidders except bidder i, then the process continues with step 216b-4. In step 216b-4, the computer assigns the quantity $\overline{Q}_k - \sum_{j\neq i} Q_k^j$ or the quantity Q_k^i—whichever of the two quantities is smaller—of objects of type k to bidder i. The process then proceeds with step 216b-5, in which the computer determines whether all types have been considered for bidder i. If not, the process loops back to step 216b-2. If all types have been considered for bidder i, the process continues with step 216b-6, in which the computer determines whether all bidders have been considered. If not, the process loops back to step 216b-1. If all bidders have been considered, the process goes to step 218 of Figure 3.

[0059] Figure 5 is a flow diagram of a subprocess of step 208. It begins with step 208-1, in which the computer selects a bidder which has submitted a bid but which has not yet been considered. This bidder is denoted bidder i. One exemplary way of selecting which bidder to consider is to select the earliest-time-stamped bid which has not yet been considered. In step 208-2, the computer recalls from memory the most recent previously-processed bid by bidder i, the bidder currently being considered. The most recent previously-processed bid is denoted $(Q_1^{i,t-1},\ldots,Q_m^{i,t-1})$. In step 208-3, the computer determines whether bidder i's current bid satisfies an eligibility rule, for example: $Q_k^{i,t} \leq Q_k^{i,t-1}$. If bidder i's current bid satisfies the eligibility rule, then the process skips to step 208-5. If bidder i's current bid does not satisfy the eligibility rule, the process continues with step 208-4, in which the computer adjusts bidder i's current bid so as to satisfy the eligibility rule. One exemplary way of doing this is to insert bidder i's most recent previously-processed bid, $(Q_1^{i,t-1},\ldots,Q_m^{i,t-1})$, as bidder i's current bid. In step 208-5, the computer determines whether bidder i's current bid satisfies the additional constraints: $\sum_{j=1}^{n} Q_k^j \geq \overline{Q}_k$, for all types $k = 1,\ldots,m$. If bidder i's current bid satisfies these additional constraints, then the process skips to step 208-7. If bidder i's current bid does not satisfy these additional constraints, the process continues with step 208-6, in which the computer adjusts bidder i's bid so as to satisfy the additional constraints. One exemplary way of doing this is to insert bidder i's most recent previously-processed bid, $(Q_1^{i,t-1},\ldots,Q_m^{i,t-1})$, as bidder i's current bid. A second exemplary way of doing this is to substitute Q_k^i with $\overline{Q}_k - \sum_{j\neq i} Q_k^j$ as bidder i's quantity for type k, for every $k = 1,\ldots,m$ violating the additional constraint. In step 208-7, the computer determines whether all bidders who have submitted bids have yet been considered. If not, the process loops back to step 208-1. If all bidders have been considered, the process goes to step 210 of Figure 3.

[0060] Another embodiment of the inventive system is described by a slightly different flow diagram, Figure 6. The difference between Figure 3 and Figure 6 is that—in Figure 6—the step 316 is deferred until the end of the auction. In that event, the auctioneer is allowed to wait until the conclusion of the auction to determine which objects were assigned at which prices in the course of the auction.

[0061] Figure 6 is thus a flow diagram of an auction process in accordance with another embodiment of the present invention. The process starts with step 302, in which memory locations at the computer are initialized. In step 302, the appropriate memory locations are initialized with information such as the number of types of objects for auction, the quantity of each type of object for auction, and the initial price vector. In step 304, the computer outputs auction information, including the current price vector $(P_1,\ldots,P_m)$. In step 306, the computer receives bids $(Q_1^i,\ldots,Q_m^i)$ from bidders. In step 308, the computer closes the bidding for the current round and processes bids. This process is shown in more detail in Figure 5 (but with "From Step 206" replaced by "From Step 306", and with "To Step 210" replaced by "To Step 310", etc.). In step 310, the computer determines whether: (a) there exist zero feasible assignments of the available quantity; (b) there exists one feasible assignment of the available quantity; or (c) there exist two or more feasible assignments of the available quantity. If, at step 310, the computer determines that (c) there exist two or more feasible assignments of the available quantity, then the process goes to step 318 in which the computer revises the current price vector $(P_1,\ldots,P_m)$ and generates the bidding history and any auction announcements and messages. One exemplary rule for revising the price is that, for every $k = 1,\ldots,m$, the price P_k of objects of type k is raised by $c[(\sum_{j=1}^{n} Q_k^{j,t})-\overline{Q}_k]$, where c is a positive constant (i.e., the price for each type is increased in direct proportion to the excess demand for that type). The process then loops.to step 304.

[0062] If, at step 310, the computer determines that (a) there exist zero feasible assignments of the available quantity, then the process goes to step 312 in which the computer assigns units in accordance with a rationing rule. One exemplary rationing rule is for the computer to honor the various bidders' attempts to decrease their bids by time priority (i.e., in the order that the bids were submitted) so long as a feasible assignment continues to exist. A second exemplary rationing rule is for the computer to increase each bidder's quantity bid in constant proportion to the bidder's most recent attempted reduction, $Q_k^{i,t+1}$ - $Q_k^{i,t}$, until one assignment becomes feasible. If, at step 310, the computer determines that (b) there exists one feasible assignment of the available quantity, then the process goes to step 314 in which the computer assigns units in accordance with the one feasible assignment. After steps 312 or 314, the process goes to step 316 in which the computer determines bidders' payments for units assigned, retroactively, in accordance with current prices at the time the units were clinched. If, at every time that units were clinched, the maximal quantity vector $(q_1^i,...,q_m^i)$ that bidder i was guaranteed to win was unique, then the determination of payments is straightforward. For the case where the sequence of maximal guaranteed quantity vectors $(q_1^i,...,q_m^i)$ for bidder i was not unique, one exemplary way for the computer to determines bidders' payments is to determine the sequence of maximal guaranteed quantity vectors consistent with the ultimate assignment of units that maximizes bidder i's payment. After step 316, the auction is deemed to have ended, and so the process ends.

[0063] Another embodiment of the inventive system is described by the same flow diagrams— except with step 216 or step 316 (and hence Figures 4a and 4b) deleted. In that event, the auction still proceeds with multiple ascending clocks, but now objects are not assigned in intermediate rounds, and so every object of a given type is assigned at the same price.

Further Embodiments with Interactions Among Different Types of Units

[0064] The embodiments of the present invention that have thus far been discussed in this document have been premised, in their logic, on an activity rule which considered each type of unit separately. Each bidder was constrained to bid a quantity on each type of unit that is no greater than any of his earlier quantities bid on that same type of unit. In notation, $Q_k^{i,t} \leq Q_k^{i,t+1}$, for every k= 1,...,m.

[0065] In many applications, it may be advantageous to instead use an activity rule which allows interactions among different types of units. One of the simplest examples is an activity rule for a bidder under which the aggregate quantity bid for all types of units is constrained to be no greater than any of his earlier aggregate quantities bid for all types of units. In notation, $\sum_{k=1}^{m} Q_k^{i,t} \leq \sum_{k=1}^{m} Q_k^{i,t+1}$.

[0066] Observe that the latter activity rule is a looser constraint on bidders than the former activity rule. For example, in Example 3 (see also Tables 2 and 5), a bid of (500, 0, 600) may be followed by a bid of (600, 300, 100) under the latter activity rule, but not under the former activity rule. Since more possible future bids exist under the latter activity rule, smaller quantities are determined to have clinched at a given bidding state when analyzed under the latter activity rule.

[0067] Activity rules that allow interactions among different types of units may be particularly advantageous in applications where substitution among the different types of units are most relevant to bidders. For example, in the application to allocation of capacity for a gas pipeline system, a bidder may be bringing in his gas by ship, and may be able to equally easily use any of four terminals. In that event, all he may really care about is his aggregate quantity summed over the four terminals (adjusted for minor cost differences among the terminals). In the application to selling Treasury securities, a bidder may be looking for a safe short-term investment, and may be able to equally easily use 3-month, 6-month or 12-month Treasury securities. Let us denote these three types of Treasury securities as Type 1, Type 2 and Type 3, respectively. In that event, all he may really care about is his aggregate quantity summed over the three times to maturity (adjusted for price differences on the yield curve).

[0068] Figure 7 is a flow diagram of an auction process in accordance with one embodiment of the present invention. The process starts with step 102, in which memory locations are initialized. In step 102, the appropriate memory locations are initialized with information such as the number of types of objects for auction, the quantity of each type of object for auction, and the initial price vector. In step 104, the bidding information processor transmits auction information, including the starting price vector $(P_1,...,P_m)$, and transmits it to bid entry terminals. In step 106, bid entry terminals receive auction information from the bidding information processor and display it to bidders. In step 108, bid entry terminals receive bids $(Q_1^i,..., Q_m^i)$ from bidders and transmit them to the bidding information processor. In step 110, the bidding information processor receives the bids transmitted from bid entry terminals and transmits confirmation messages. In step 112, the bidding information processor closes the bidding for the current round and processes bids received from bid entry terminals. This process is shown in more detail in Figure 9. In step 114, the bidding information processor assigns objects, if any, at the current prices. This process is shown in more detail in Figures 8a, 8b and 8c. In step 116, the bidding information processor determines if any objects remain unassigned. If so, the process goes to step 118 in which the bidding information processor increments the current price vector and generates the bidding history and any auction announcements and messages. One exemplary rule for incrementing the price is that, for every $k = 1,...,m$, the price P_k of objects of type k is raised by c $[(\sum_{i=1}^{n} Q_k^{i,t})-\bar{Q}_k]$, where c is a positive constant (i.e., the price for each type is increased in direct proportion to the excess demand for that type). The process then loops to step 104. If no objects remain unassigned, then the process ends.

[0069] Figure 8a is a flow diagram of a subprocess of step 114. It begins with step 114-1, in which the bidding information processor sums the quantities demanded by all the bidders and for all the types of objects; also sums the quantities remaining unassigned of all the types of objects; and computes the difference between the two sums. In step 114-2, the bidding information processor determines whether the difference between these two sums is (strictly) greater than zero. If the difference between the two sums is (strictly) greater than zero, the process continues with step 114-3, in which the bidding information processor considers each bidder separately and determines an assignment of objects at the current prices. This step is shown in more detail in Figure 8b. The process then goes to step 116 of Figure 7. If the difference between the two sums is not (strictly) greater than zero, the process continues with step 114-4, in which each bidder is assigned the quantity bid for each type of object at the current price, and since no objects remain unassigned, the auction ends after proceeding to step 116 of Figure 7.

[0070] Figure 8b is a flow diagram of a subprocess of step 114-3. It begins with step 114-3-1, in which a bidder which has not yet been considered is selected. In step 114-3-2, for the bidder currently being considered, the bidding information processor sums the quantities remaining unassigned of all the types of objects; also it sums the quantities demanded by all the bidders other than the current bidder and for all the types of objects; and it computes the difference between the two sums. The difference is denoted $\hat{Q}^i$. In step 114-3-3, the bidding information processor determines whether $\hat{Q}^i$ is (strictly) greater than zero. If $\hat{Q}^i$ is not (strictly) greater than zero, no objects are assigned to the current bidder, and the process proceeds directly to step 114-3-6. If $\hat{Q}^i$ is (strictly) greater than zero, the process continues with step 114-3-4, in which the bidding information processor considers each type of object separately and determines an assignment of objects at the current prices. This step is shown in more detail in Figure 8c. In step 114-3-5, the bidding information processor subtracts the assigned quantities from the bids of the current bidder and from the quantities for auction. In step 114-3-6, it is determined whether all bidders have been considered. If not, the process loops back to step 114-3-1. If all bidders have been considered, the process goes to step 116 of Figure 7.

[0071] Figure 8c is a flow diagram of a subprocess of step 114-3-4. It begins with step 114-3-4-1, in which for every type $k = 1,...,m$ of object, the bidding information processor sums the quantity demanded of type k by all the bidders other than the current bidder, and subtracts this from the quantity remaining unassigned of type k. The difference is denoted $\hat{Q}_k^i$. In step 114-3-4-2, the bidding information processor determines whether the sum of the $\hat{Q}_k^i$, summed over all types $k = 1,...,m$ of objects, equals $\hat{Q}^i$. If the sum of the $\hat{Q}_k^i$ equals $\hat{Q}^i$, then the process continues with step 114-3-3. In step 114-3-3, $(\hat{Q}_1^i,..., \hat{Q}_m^i)$ is determined to be the assignment to bidder i, and the process goes to step 114-3-5 of Figure 8b. If the sum of the $\hat{Q}_k^i$ does not equal $\hat{Q}^i$, then the process continues with step 114-3-4. In step 114-3-4, the bidding information processor determines the most-preferred assignment $(Q_1^r,...,Q_m^r)$, which satisfies the following constraints: $0 \le Q_k^r \le \hat{Q}_k^i$ for every type $k = 1,..., m$, and $\sum_{k=1}^{m} Q_k^r = \hat{Q}^i$. One exemplary way of selecting the most-preferred assignment is to select the $(Q_1^r,...,Q_m^r)$ consistent with the constraints such that the difference between the expected final price vector and the current price vector, multiplied by $(Q_1^r,...,Q_m^r)$ as a dot product, is minimized. The determined $(Q_1^r,...,Q_m^r)$ is deemed to be the assignment to bidder i, and the process goes to step 114-3-5 of Figure 8b.

[0072] Figure 9 is a flow diagram of a subprocess of step 112. It begins with step 112-1, in which the bidding information processor considers a bidder who has submitted a bid which has not yet been considered. This bidder is denoted bidder i. One exemplary way of selecting which bidder to consider is to select the earliest-time-stamped bid which has not yet been considered. In step 112-2, the bidding information processor recalls from memory the most recent previously-processed bid by bidder i, the bidder currently being considered. The previously-processed bid is denoted $(Q_1^{i,t-1},...,Q_m^{i,t-1})$. In step 112-3, the bidding information processor determines whether bidder *i*s current bid satisfies the eligibility rule: $\sum_{k=1}^{m} Q_k^{i,t} \le \sum_{k=1}^{m} Q_k^{i,t-1}$. If bidder *i*s current bid satisfies the eligibility rule, then the process skips to step 112-5. If bidder *i*s current bid does not satisfy the eligibility rule, the process continues with step 112-4, in which the bidding information processor adjusts bidder *i*s bid so as to satisfy the eligibility rule. One exemplary way of doing this is to insert bidder *i*s most recent previously-processed bid, $(Q_1^{i,t-1},...,Q_m^{i,t-1})$, as bidder *i*s current bid. In step 112-5, the bidding information processor determines whether bidder *i*s current bid satisfies the additional constraints: $\sum_{j=1}^{n} Q_k^j \ge \bar{Q}_k$, for all types $k = 1,...,m$. If bidder *i*s current bid satisfies these additional constraints, then the process skips to step 112-7. If bidder *i*s current bid does not satisfy these additional constraints, the process continues with step 112-6, in which the bidding information processor adjusts bidder *i*s bid so as to satisfy the additional constraints. One exemplary way of doing this is to insert bidder *i*s most recent previously-processed bid, $(Q_1^{i,t-1},...,Q_m^{i,t-1})$, as bidder *i*s current bid. A second exemplary way of doing this is to substitute Q_k^i with $\bar{Q}_k-\sum_{j\neq i} Q_k^j$ as bidder *i*s quantity for type k, for every $k = 1,...,m$ violating the additional constraint, provided that this substitution does not lead the eligibility rule to be violated. In step 112-7, the bidding information processor determines whether all bidders who have submitted bids have yet been considered. If not, the process loops back to step 112-1. If all bidders have been considered, the process goes to step 114 of Figure 7.

[0073] Another embodiment of the inventive system is described by the same flow diagrams— except with Step 114-3-4-4 deferred until the end of the auction. In that event, the auctioneer is allowed to wait until the conclusion of the auction to determine which objects were assigned at which prices in the course of the auction.

[0074] Another embodiment of the inventive system is described by the same flow diagrams— except with Step 114-3 (and hence Figures 8b and 8c) deleted. In that event, the auction still proceeds with multiple ascending clocks, but now objects are not assigned in intermediate rounds, and so every object of a given type is assigned at the same price.

Claims

1. A method for using a computer to implement an auction of heterogeneous objects, two or more bidders participating in the auction, the auction allowing assignment of objects of the same type to one or more of the bidders at different prices, the method comprising:

   a) inputting, into the computer, bids from participating bidders;
   b) determining at the computer, based on the bids, whether there is at least one object which is desired by only one bidder and,
   c) in the event there is at least one object which is desired by only one bidder, assigning the determined object or objects to the determined bidder,

   wherein said determining comprises determining if a nonempty set of objects is minimal in the winning set of a bidder.

2. A method as recited in claim 1 in which inputting and determining steps are repetitively performed until all objects are assigned, wherein any bidder is constrained in bidding based on prior bids.

3. A method as recited in claim 1 wherein the determining comprises:

   b11) selecting a bidder,
   b12) selecting an object type,
   b13) determining if bids of other bidders for this object type total less than available objects of this type, in that event, and the assigning comprises assigning to the selected bidder the smaller of, the difference between number of available objects and sum of bids by other bidders or the selected bidders bid for objects of this selected type.

4. A method as recited in claim 3 which includes the further steps of:

   d) validating bids by

   d1) selecting a bidder,
   d2) comparing a most recent bid by the bidder to an earlier bid of the same bidder, and
   d3) determining for each object type if a most recent bid carries a quantity in excess of an earlier bid,
   d4) in the event the determination of step c3) is positive, adjusting the most recent bid, or
   d5) repeating steps d1)-c4) for another bidder.

5. A method for using a computer to implement an auction of heterogeneous objects, two or more bidders participating in the auction, the auction allowing assignment of objects of the same type to one or more of the bidders at different prices, the method comprising:

   a) inputting, into the computer, bids from participating bidders;

   b) determining at the computer, based on the bids, whether there is at least one object which is desired by only one bidder and, if so, assigning the determined object or objects to the determined bidder, and

   c) repeating steps a) and b) until no objects remain unassigned.

6. A method as recited in claim 5 wherein said determining includes:

   b1) selecting a bidder,
   b2) summing, for all other bidders, a first sum representing a quantity of all objects bid for by the other bidders,
   b3) identifying a quantity of all unassigned objects and determining if the quantity is strictly greater than the first sum,
   b4) in the event the quantity is strictly greater than the first sum, then
   b5) assigning objects either based on the bidder's bid or using a most preferred assignment rule, and
   b6) repeating steps b2) to b5) for each other bidder.

7. A computer system implementing an auction of heterogeneous objects, two or more bidders participat-

ing in the auction, the auction allowing assignment of objects of the same type to one or more of the bidders at different prices, the computer system including

a) means for inputting to the computer, bids from participating bidders;

b) determining means for determining, based on the bids, whether there is at least one object which is desired by only one bidder and,

c) assigning means for assigning the determined object or objects to the determined bidder in the event there is at least one object which is desired by only one bidder.

8. The computer system of clam 7 wherein the determining means identifies each object which is desired by only one bidder and in which the assigning means assigns each such object identified by the determining means.

9. The system of claim 8 wherein the determining means includes:

b1) means for selecting a bidder,
b2) means for summing, for all other bidders, a first sum representing a quantity of all objects bid for by the other bidders,
b3) means for identifying a quantity of all unassigned objects and determining if the quantity is strictly greater than the first sum,

and wherein the assigning means includes
c1) responsive means, in the event the quantity is strictly greater than the first sum, for assigning objects either based on the bidder's bid or using a most preferred assignment rule.

10. A computer system implementing an auction of heterogeneous objects, two or more bidders participating in the auction, the auction allowing assignment of objects of the same type to one or more of the bidders at different prices, the computer system including:

a) input means for inputting bids from participating bidders;
b) determining means for determining, based on the bids, whether there is at least one object which is desired by only one bidder and,
c) assigning means for assigning, in the event there is at least one object which is desired by only one bidder, the determined object or objects to the determined bidder,

where said determining means determines if a nonempty set of objects is minimal in the winning set of a bidder.

11. The computer system of claim 10 wherein the determining means identifies each object which is desired by only one bidder and in which the assigning means assigns each such object identified by the determining means.

12. The computer system of claim 10 wherein the determining means includes:

b1) means for selecting a bidder,
b2) means for selecting an object type,
b3) summing means for determining if bids of other bidders for this object type total less than available objects of this type,

and wherein the assigning means comprises
c1) responsive means, in the event the bids of other bidders for this object type totals less than the number of available objects of this type, for assigning to the selected bidder the smaller of, the difference between number of available objects and the total of bids by other bidders or the selected bidders bid for objects of this selected type.

13. The computer system as recited in claim 12 which further includes means for validating bids comprising

d1) means for selecting a bidder,
d2) measn for comparing a most recent bid by the bidder to an earlier bid of the same bidder, and
d3) means for determining, for each object type if a most recent bid carries a quantity in excess of an earlier bid, and
d4) adjusting means for adjusting the most recent bid in the event the most recent bid cartries a quantity in excess of an earlier bid.

FIG. 1

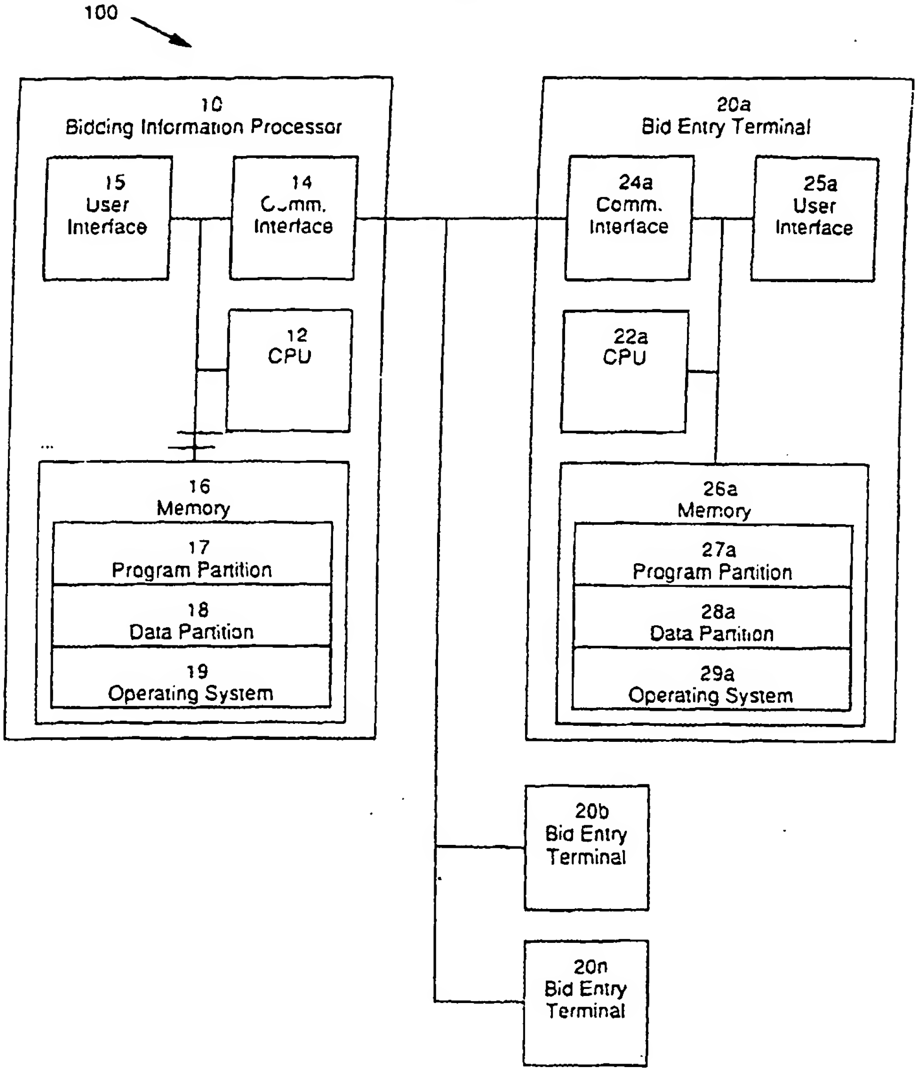

100
10
Bidding Information Processor
15
User
Interface
14
Comm.
Interface
12
CPU
16
Memory
17
Program Partition
18
Data Partition
19
Operating System
20a
Bid Entry Terminal
24a
Comm.
Interface
25a
User
Interface
22a
CPU
26a
Memory
27a
Program Partition
28a
Data Partition
29a
Operating System
20b
Bid Entry
Terminal
20n
Bid Entry
Terminal

FIG. 2

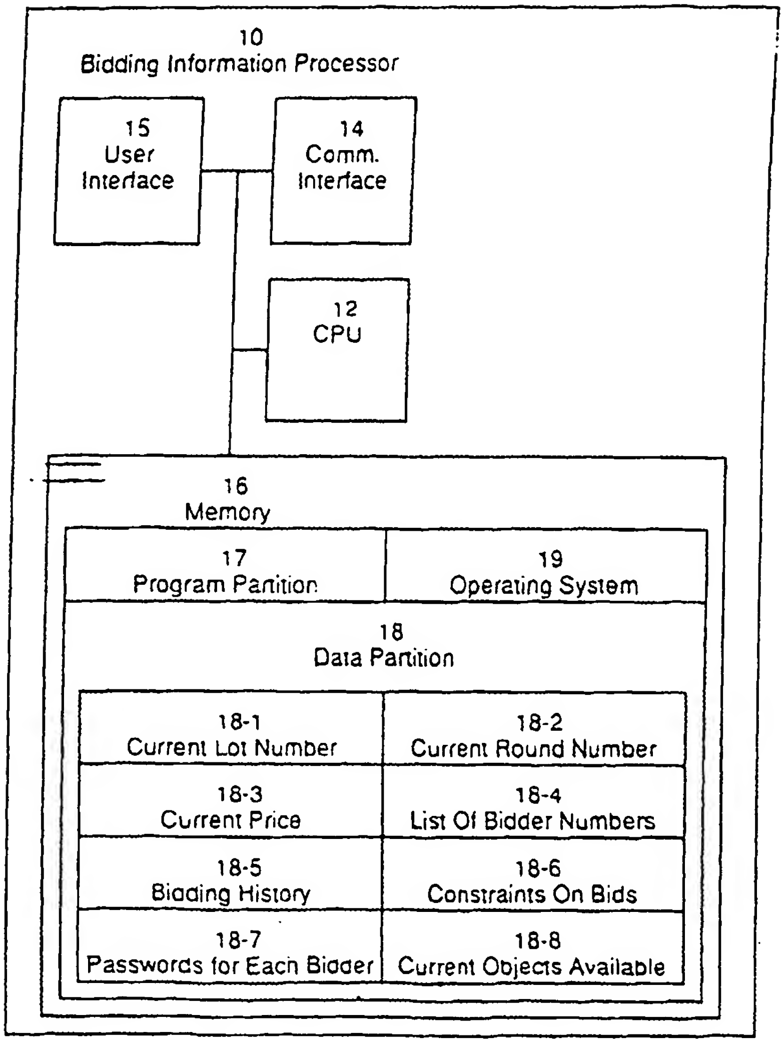

10
Bidding Information Processor
15
User
Interface
14
Comm.
Interface
12
CPU
16
Memory
17
Program Partition
19
Operating System
18
Data Partition
18-1
Current Lot Number
18-2
Current Round Number
18-3
Current Price
18-4
List Of Bidder Numbers
18-5
Bidding History
18-6
Constraints On Bids
18-7
Passwords for Each Bidder
18-8
Current Objects Available

FIGURE 3

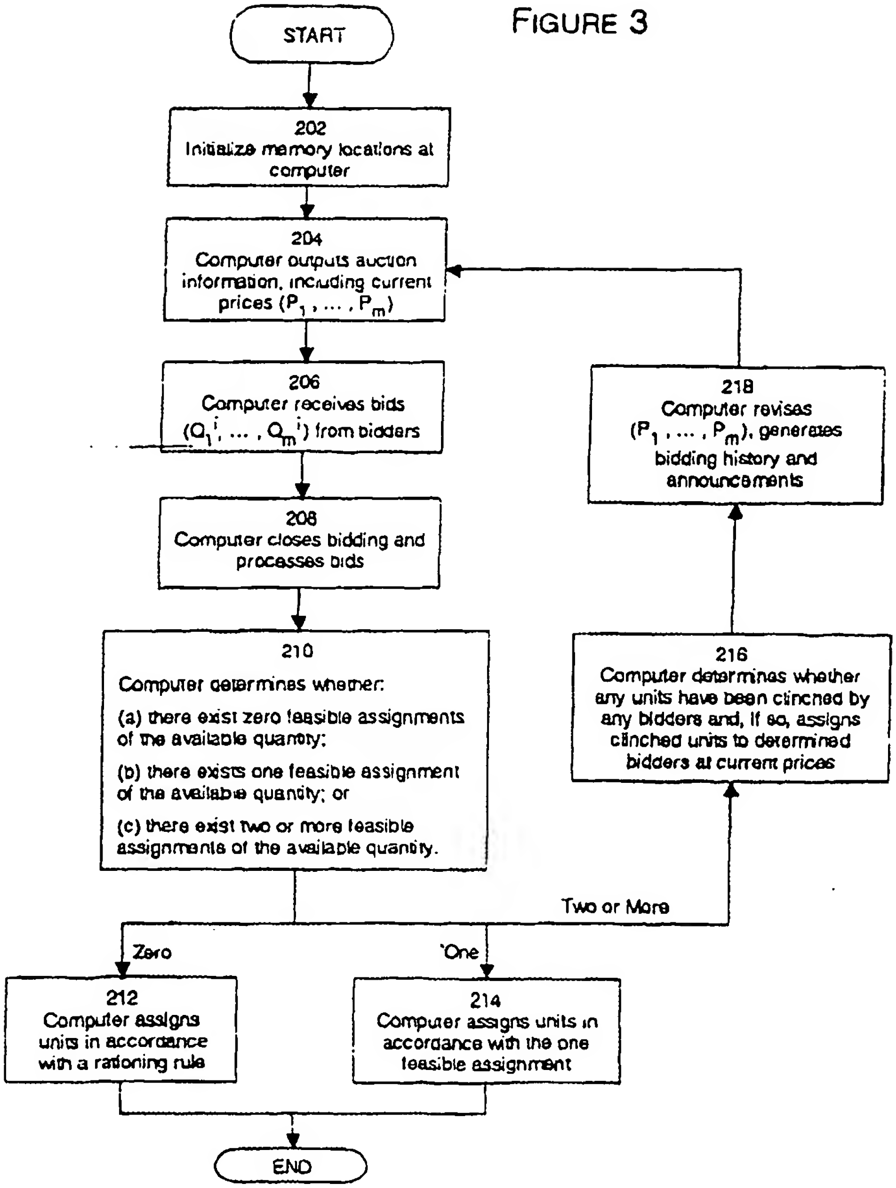

START
202
Initialize memory locations at computer
204
Computer outputs auction information, including current prices (P1 , ... , Pm)
206
Computer receives bids (Q1i, ... , Qmi) from bidders
208
Computer closes bidding and processes bids
210
Computer determines whether:
(a) there exist zero feasible assignments of the available quantity;
(b) there exists one feasible assignment of the available quantity; or
(c) there exist two or more feasible assignments of the available quantity.
Zero
One
Two or More
212
Computer assigns units in accordance with a rationing rule
214
Computer assigns units in accordance with the one feasible assignment
216
Computer determines whether any units have been clinched by any bidders and, if so, assigns clinched units to determined bidders at current prices
218
Computer revises (P1 , ... , Pm), generates bidding history and announcements
END

FIGURE 4a

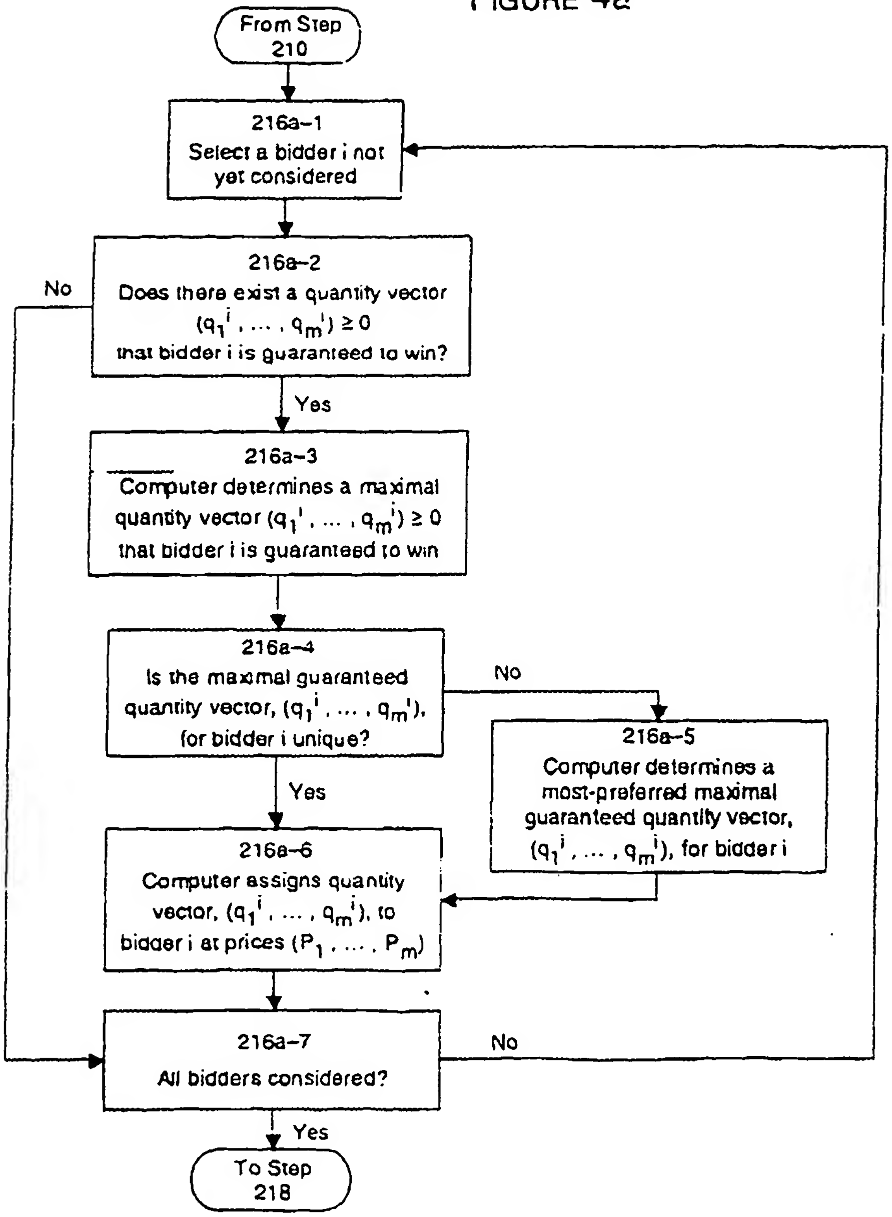

From Step 210
216a-1
Select a bidder i not yet considered
216a-2
Does there exist a quantity vector $(q_1^i, \ldots, q_m^i) \geq 0$ that bidder i is guaranteed to win?
No
Yes
216a-3
Computer determines a maximal quantity vector $(q_1^i, \ldots, q_m^i) \geq 0$ that bidder i is guaranteed to win
216a-4
Is the maximal guaranteed quantity vector, $(q_1^i, \ldots, q_m^i)$, for bidder i unique?
No
216a-5
Computer determines a most-preferred maximal guaranteed quantity vector, $(q_1^i, \ldots, q_m^i)$, for bidder i
Yes
216a-6
Computer assigns quantity vector, $(q_1^i, \ldots, q_m^i)$, to bidder i at prices $(P_1, \ldots, P_m)$
216a-7
All bidders considered?
No
Yes
To Step 218

FIGURE 4b

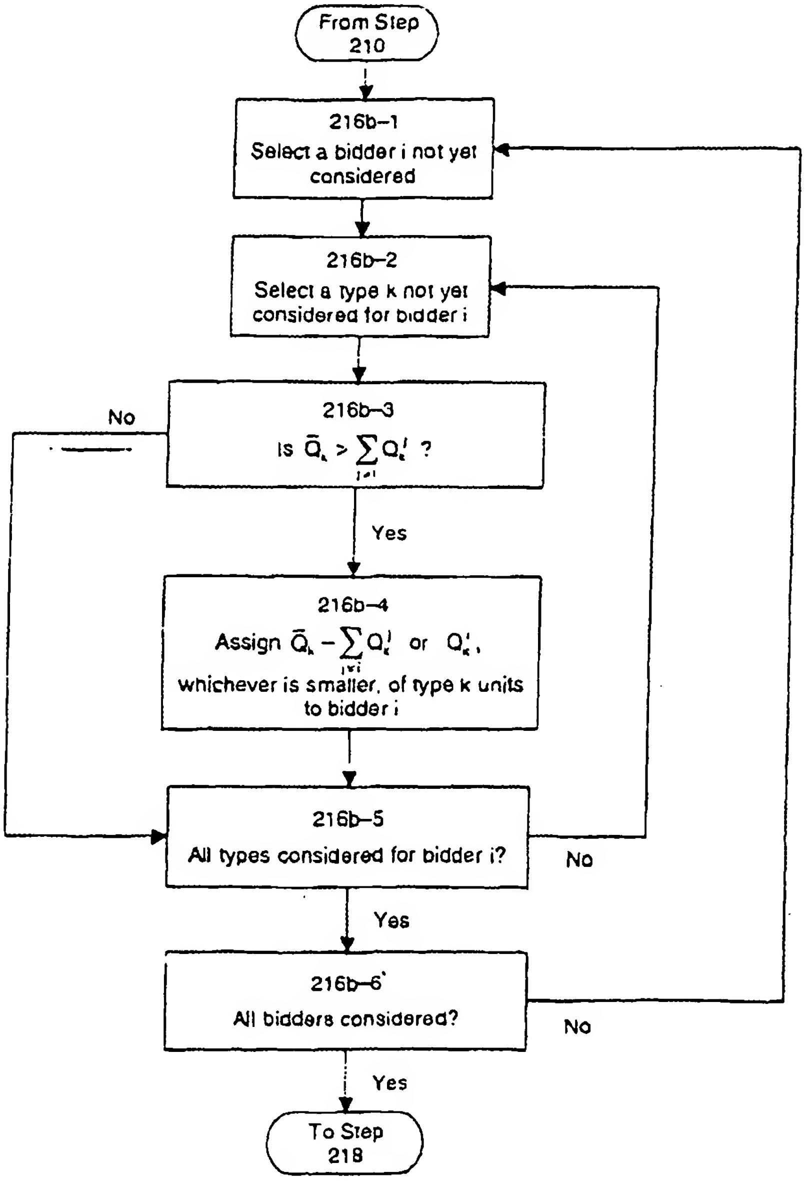

From Step 210
216b–1
Select a bidder i not yet considered
216b–2
Select a type k not yet considered for bidder i
216b–3
Is $\bar{Q}_k > \sum_{j \neq i} Q_k^j$?
No
Yes
216b–4
Assign $\bar{Q}_k - \sum_{j \neq i} Q_k^j$ or Q_k^i, whichever is smaller, of type k units to bidder i
216b–5
All types considered for bidder i?
No
Yes
216b–6
All bidders considered?
No
Yes
To Step 218

FIGURE 5

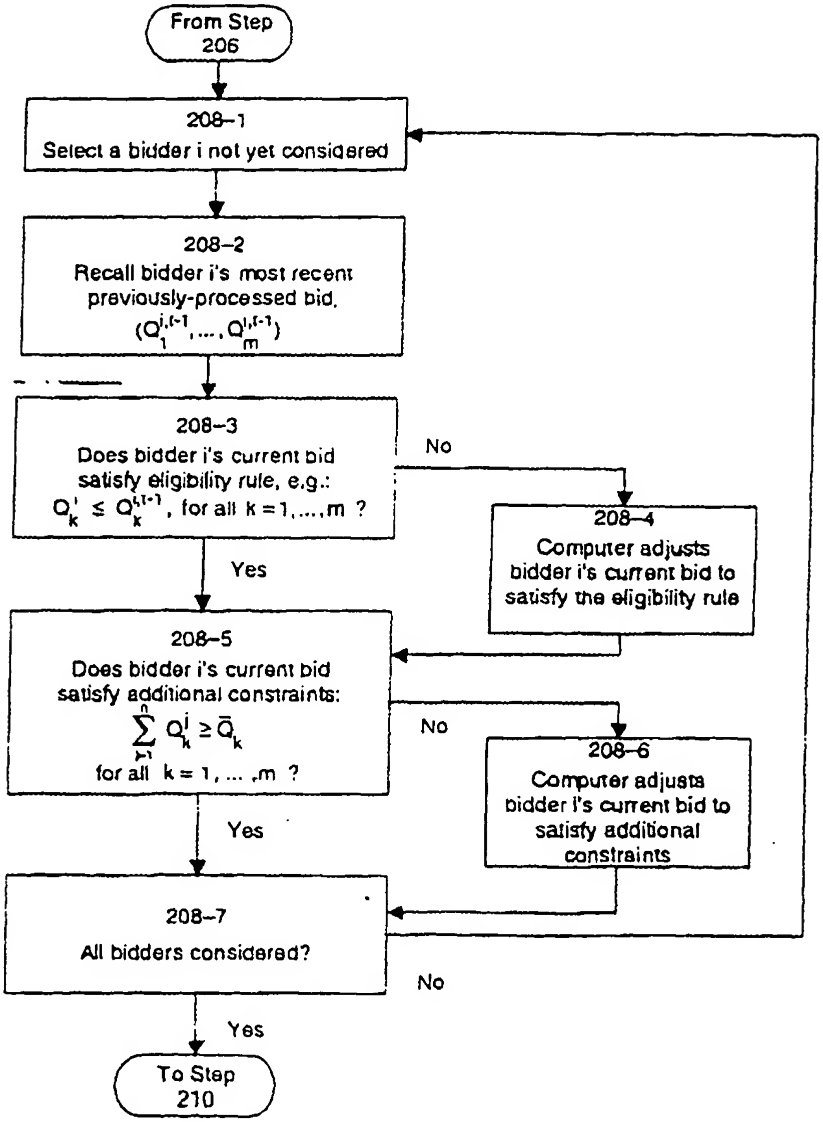

From Step 206
208-1
Select a bidder i not yet considered
208-2
Recall bidder i's most recent previously-processed bid.
$(Q_1^{i,t-1}, \ldots, Q_m^{i,t-1})$
208-3
Does bidder i's current bid satisfy eligibility rule, e.g.:
$Q_k^i \le Q_k^{i,t-1}$, for all $k = 1, \ldots, m$?
No
208-4
Computer adjusts bidder i's current bid to satisfy the eligibility rule
Yes
208-5
Does bidder i's current bid satisfy additional constraints:
$\sum_{i=1}^{n} Q_k^i \ge \bar{Q}_k$
for all $k = 1, \ldots, m$?
No
208-6
Computer adjusts bidder i's current bid to satisfy additional constraints
Yes
208-7
All bidders considered?
No
Yes
To Step 210

FIGURE 6

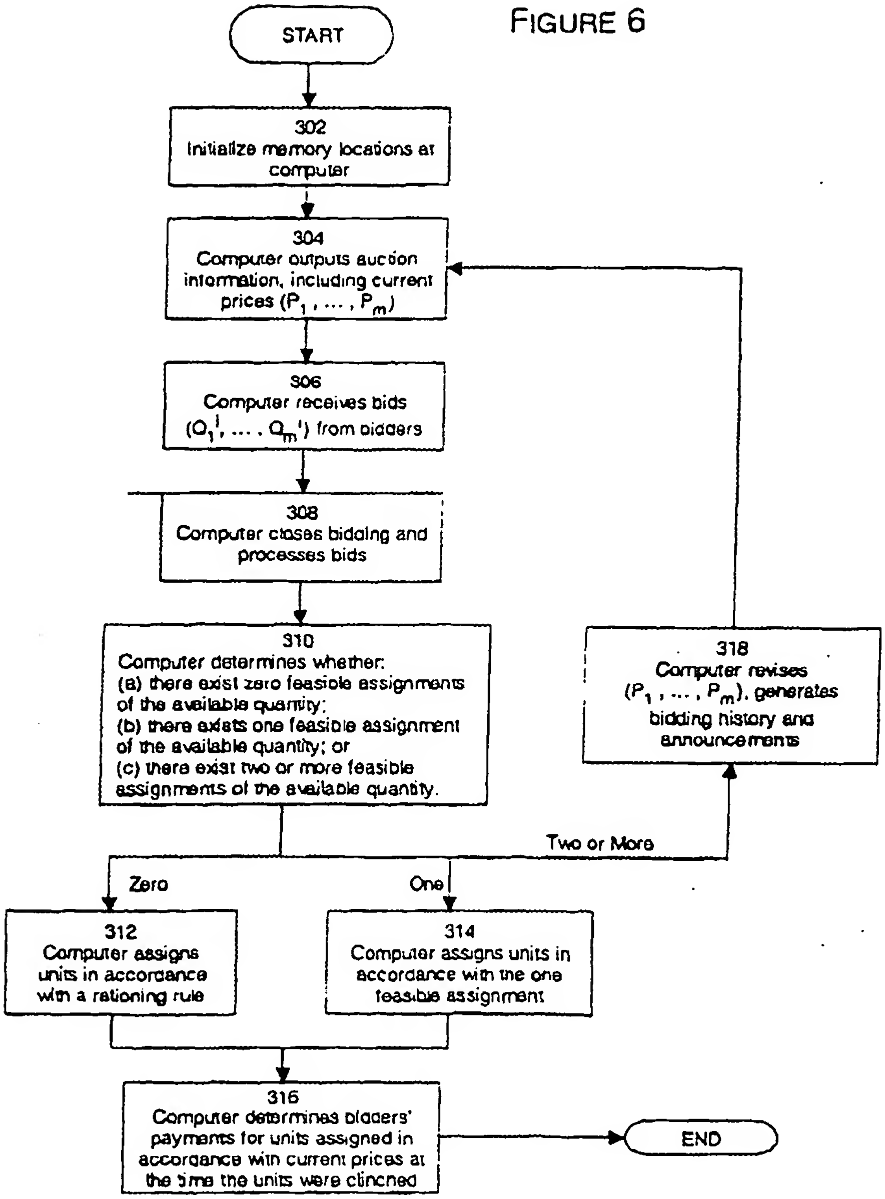
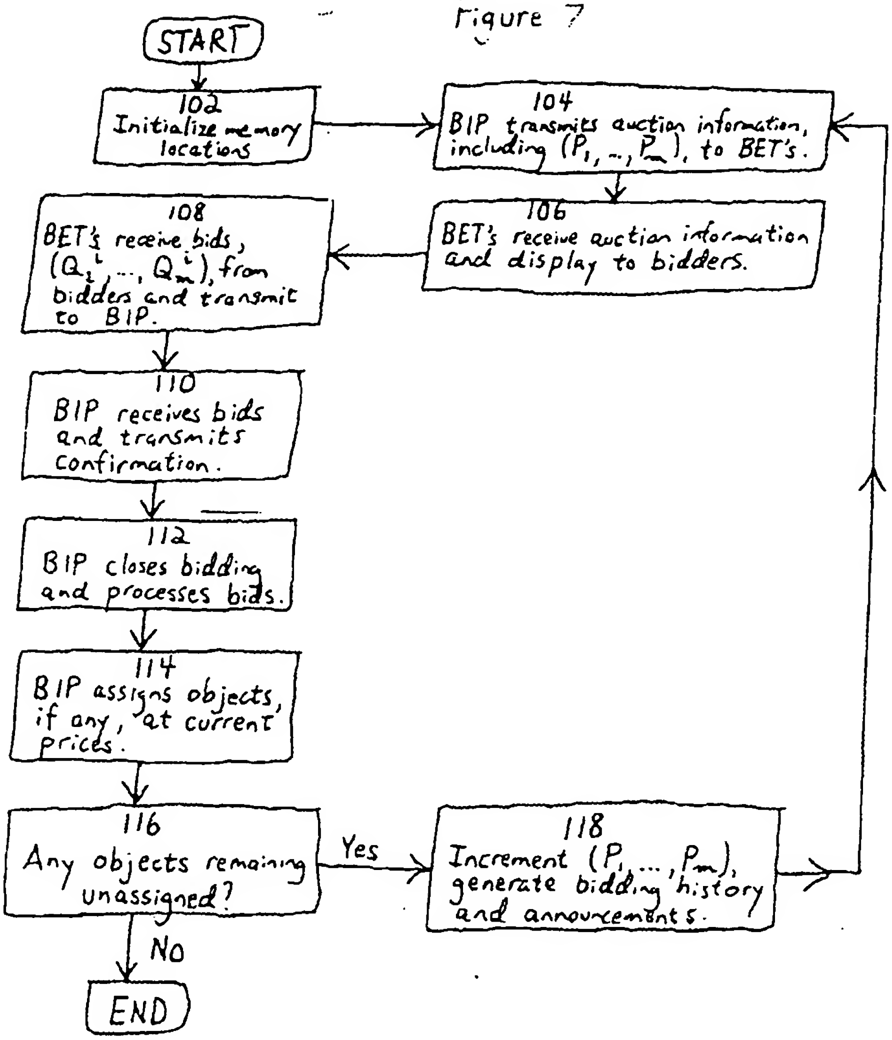

START
302
Initialize memory locations at computer
304
Computer outputs auction information, including current prices (P_1, ... , P_m)
306
Computer receives bids (Q_1^i, ... , Q_m^i) from bidders
308
Computer closes bidding and processes bids
310
Computer determines whether:
(a) there exist zero feasible assignments of the available quantity;
(b) there exists one feasible assignment of the available quantity; or
(c) there exist two or more feasible assignments of the available quantity.
Two or More
Zero
One
312
Computer assigns units in accordance with a rationing rule
314
Computer assigns units in accordance with the one feasible assignment
316
Computer determines bidders' payments for units assigned in accordance with current prices at the time the units were clinched
END
318
Computer revises (P_1, ... , P_m), generates bidding history and announcements

Figure 7


START
102
Initialize memory locations
104
BIP transmits auction information, including (P_1, .., P_m), to BET's.
106
BET's receive auction information and display to bidders.
108
BET's receive bids, (Q_1^i, ..., Q_m^i), from bidders and transmit to BIP.
110
BIP receives bids and transmits confirmation.
112
BIP closes bidding and processes bids.
114
BIP assigns objects, if any, at current prices.
116
Any objects remaining unassigned?
Yes
118
Increment (P_1, ..., P_m), generate bidding history and announcements.
No
END

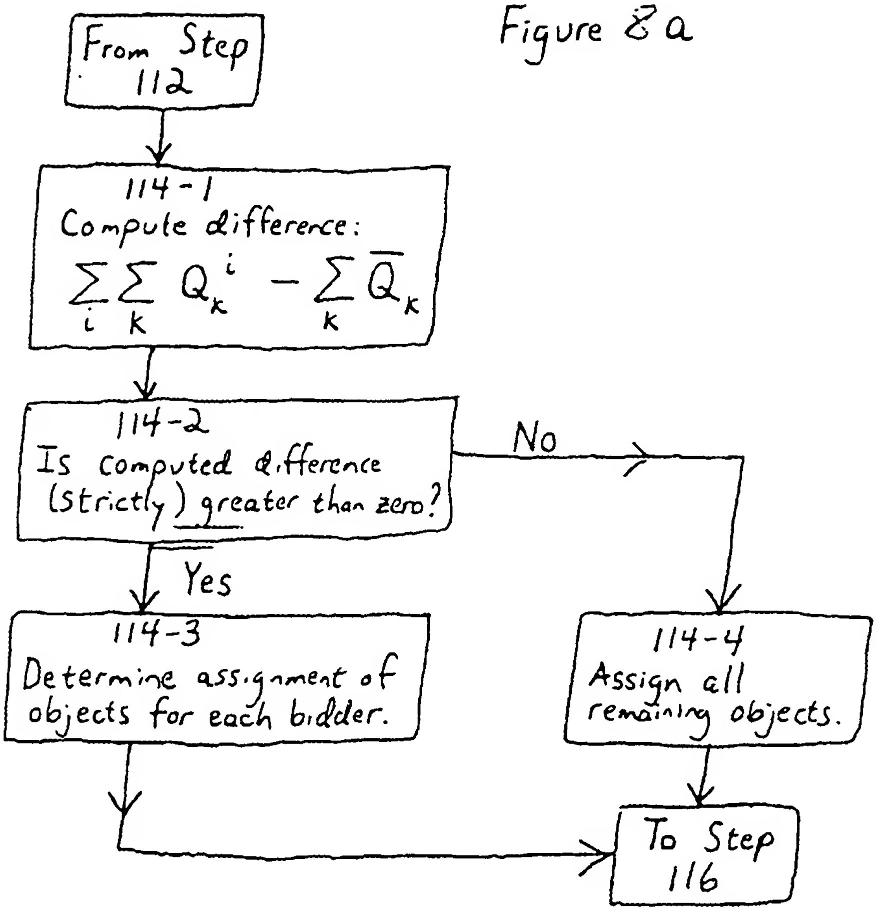


Figure 8a
From Step 112
114-1
Compute difference:
$\sum_i \sum_k Q_k^i - \sum_k \overline{Q}_k$
114-2
Is computed difference (strictly) greater than zero?
No
Yes
114-3
Determine assignment of objects for each bidder.
114-4
Assign all remaining objects.
To Step 116

Figure 8b

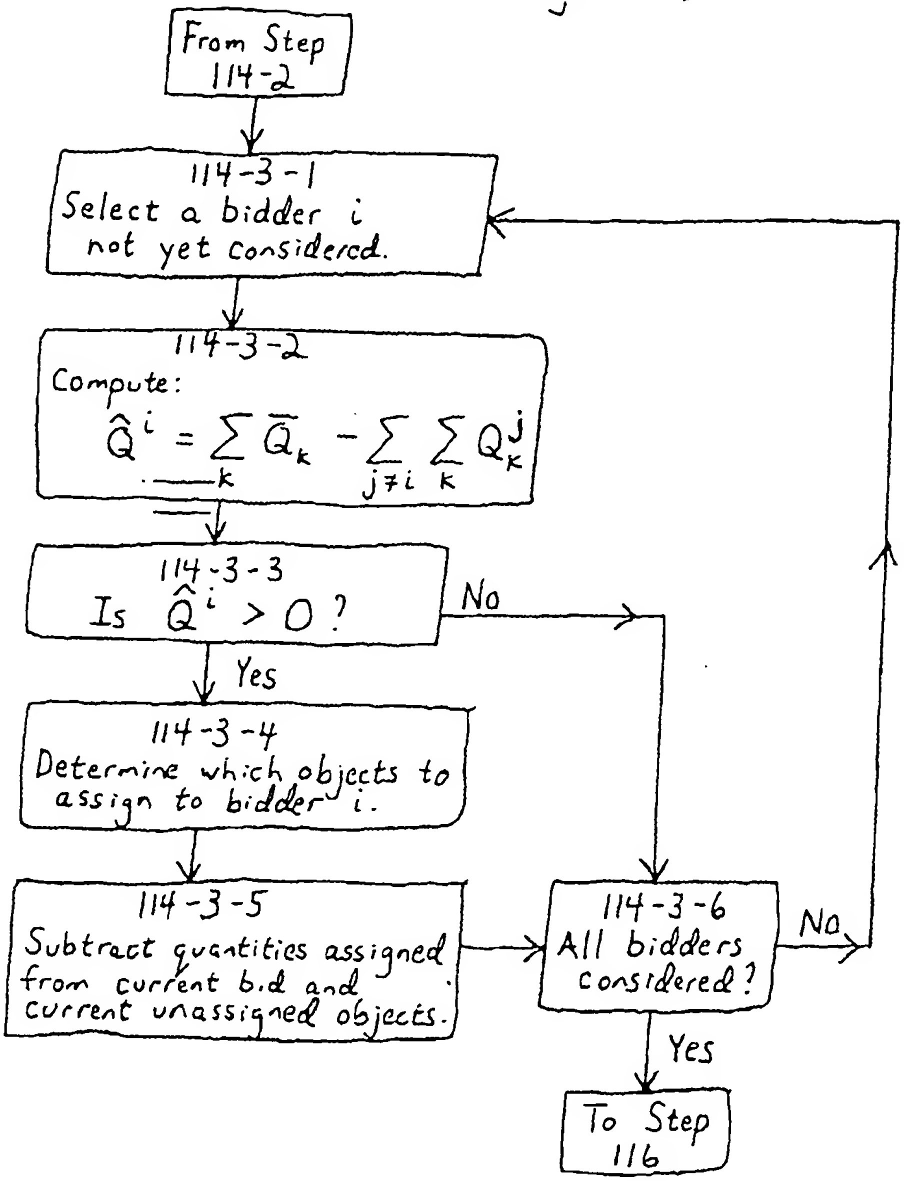

From Step 114-2
114-3-1
Select a bidder i not yet considered.
114-3-2
Compute:
$\hat{Q}^i = \sum_k \bar{Q}_k - \sum_{j \neq i} \sum_k Q_k^j$
114-3-3
Is $\hat{Q}^i > 0$?
No
Yes
114-3-4
Determine which objects to assign to bidder i.
114-3-5
Subtract quantities assigned from current bid and current unassigned objects.
114-3-6
All bidders considered?
No
Yes
To Step 116

Figure 8c

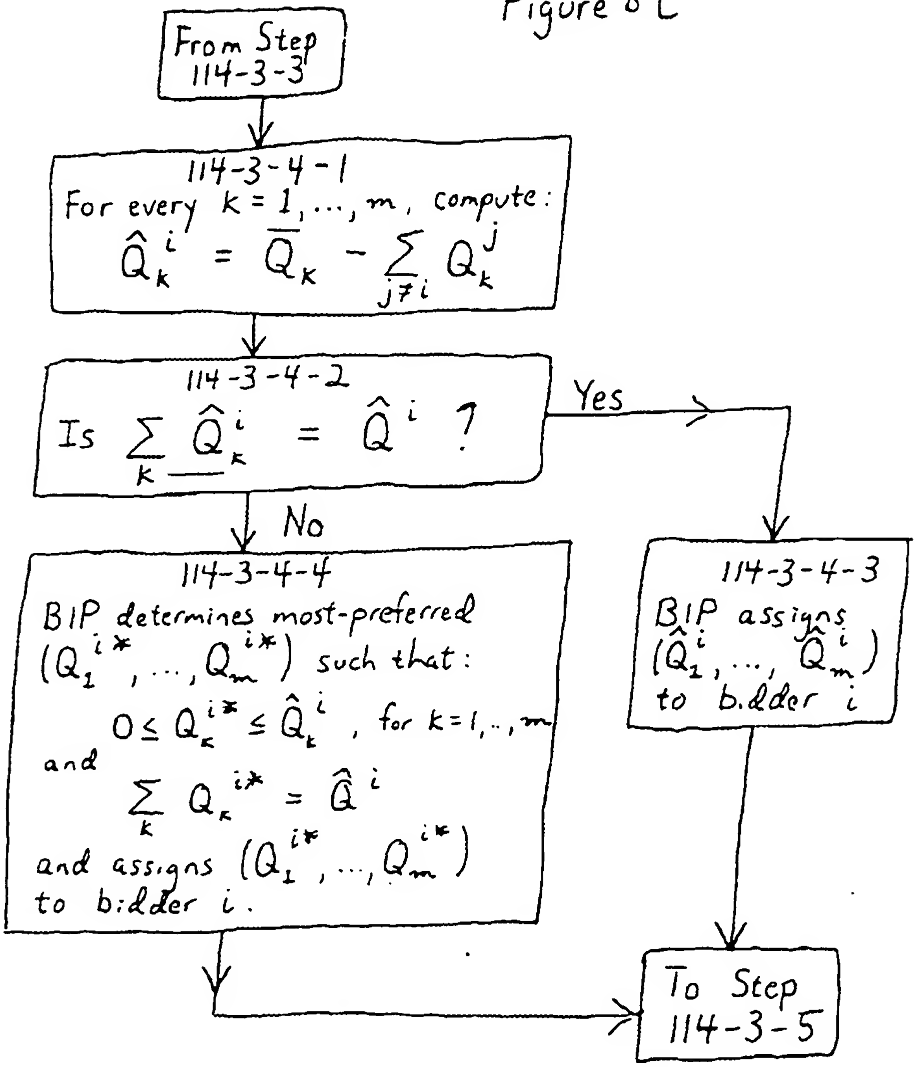
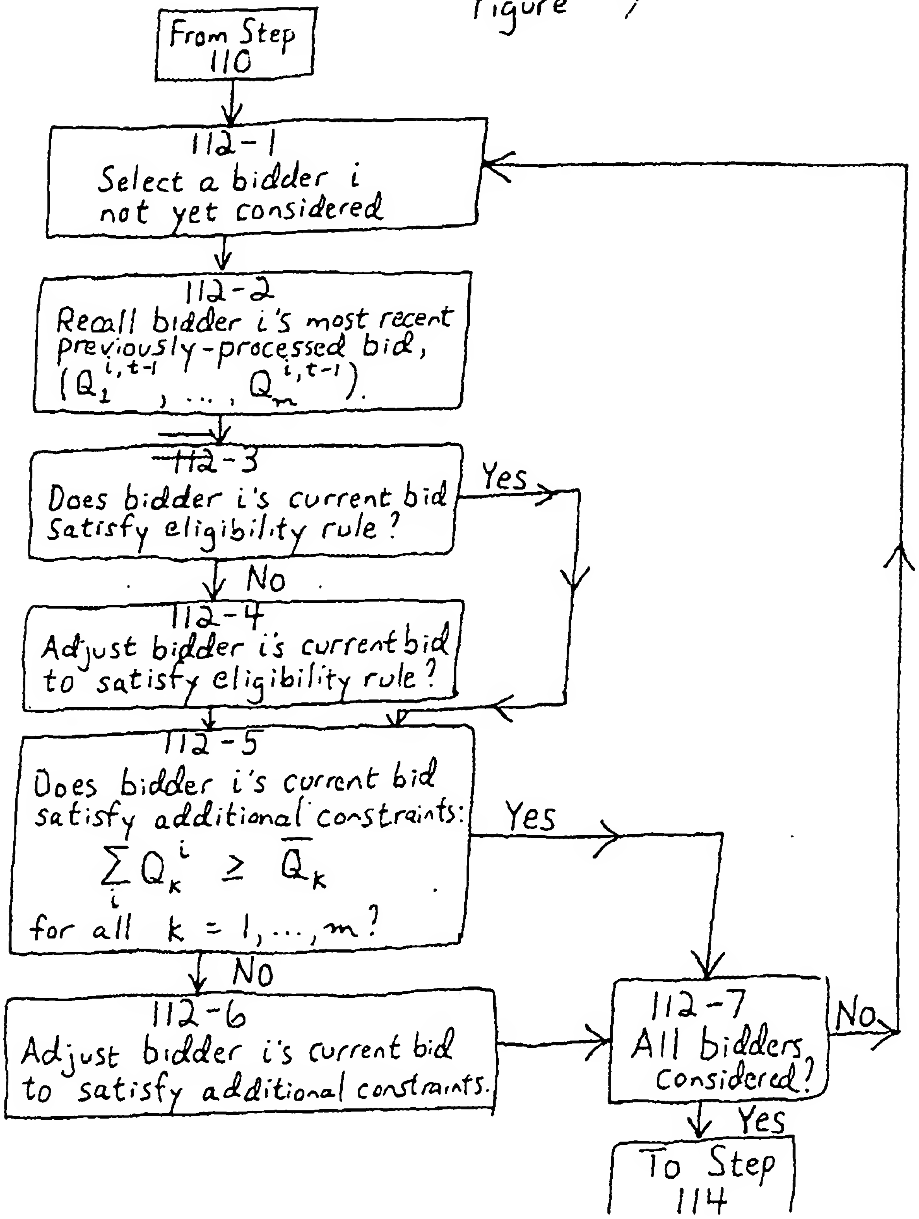

From Step 114-3-3
114-3-4-1
For every $k = 1, \ldots, m$, compute:
$\hat{Q}_k^i = \overline{Q}_k - \sum_{j \neq i} Q_k^j$
114-3-4-2
Is $\sum_k \hat{Q}_k^i = \hat{Q}^i$?
Yes
No
114-3-4-4
BIP determines most-preferred $(Q_1^{i*}, \ldots, Q_m^{i*})$ such that:
$0 \leq Q_k^{i*} \leq \hat{Q}_k^i$, for $k = 1, \ldots, m$
and
$\sum_k Q_k^{i*} = \hat{Q}^i$
and assigns $(Q_1^{i*}, \ldots, Q_m^{i*})$ to bidder i.
114-3-4-3
BIP assigns $(\hat{Q}_1^i, \ldots, \hat{Q}_m^i)$ to bidder i
To Step 114-3-5

Figure 9


From Step 110
112-1
Select a bidder i not yet considered
112-2
Recall bidder i's most recent previously-processed bid, $(Q_1^{i,t-1}, \ldots, Q_m^{i,t-1})$.
112-3
Does bidder i's current bid satisfy eligibility rule?
Yes
No
112-4
Adjust bidder i's current bid to satisfy eligibility rule?
112-5
Does bidder i's current bid satisfy additional constraints: $\sum_i Q_k^i \geq \overline{Q}_k$ for all $k = 1, \ldots, m$?
Yes
No
112-6
Adjust bidder i's current bid to satisfy additional constraints.
112-7
All bidders considered?
No
Yes
To Step 114